LMU
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Gewöhnliche Differentialgleichungen

Blatt 3

Aufgabe 1. Let $x, y \in \mathbb{S}^2$ such that $y \neq x$ and $y \neq -x$.

(i) If $u \in \mathbb{S}^2$ is orthogonal to x, then the path $\sigma_{x,u} : \mathbb{R} \to \mathbb{S}^2$ parametrises the great circle $\langle\{x, u\}\rangle \cap \mathbb{S}^2$, where $\langle\{x, u\}\rangle$ is the linear span of x and u.

(ii) There is a C^∞ path $\sigma_{x,y} : [0, |y - x|] \to \mathbb{S}^2$ that parametrises the arc of the unique great circle from x to y.

Hint: For (ii) use the vector

$$u := \frac{y - \langle y, x\rangle x}{|y - \langle y, x\rangle x|}.$$

Aufgabe 2. Let $U \subseteq \mathbb{R}^n$ be path-connected and open, and let $F : U \to \mathbb{R}^n$ be a continuous vector field on U. The following are equivalent.

(i) F is conservative.

(ii) The path integral of F between any two points of U is independent of the path connecting them.

(iii) The path integral of F along any loop in U is equal to 0.

Aufgabe 3. Complete the proof of the implication (i) $\Rightarrow$ (iii) of Proposition 1.2.15.

Aufgabe 4. Let $s(t)$ be a solution curve of $\ddot{x}(t) = m^{-1}F(x(t))$, where $F(x)$ is the Newtonian gravitational field on $U_0^{(2)}$, and h is non-zero along $s(t)$.

(i) The kinetic energy T along $s(t)$ satisfies the following formula:

$$T = \frac{1}{2}\frac{h^2}{m}\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right].$$

(ii) Along $s(t)$ the functions u, θ and E satisfy the following ode:

$$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2m}{h^2}(E + u).$$

(iii) Along $s(t)$ the functions u and θ satisfy the following ode:

$$\frac{d^2u}{d\theta^2} + u = \frac{m}{h^2}.$$

Abgabe. Donnerstag, 03. Mai 2018 in der Vorlesung.

Besprechung. Montag, 28. Mai 2018, in der Übung.