LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Dr. Iosif Petrakis
N. Köpp

Sommersemester 18
19.04.2018

Gewöhnliche Differentialgleichungen

Blatt 2

Aufgabe 1. Let U be a bounded, non-empty, open, and convex subset of $\mathbb{R}^n$, such that $-u \in U$, if $u \in U$. The following hold:
(i) $0 \in U$.
(ii) If $x \in \mathbb{R}^n$, and

$$\tau_U(x) := \left\{ \lambda > 0 \mid \frac{x}{\lambda} \in U \right\},$$

then $\inf \tau_U(x)$ exists.
(iii) The function

$$||x||_U := \inf \tau_U(x)$$

is a norm on $\mathbb{R}^n$.
(iv) $U \subseteq \mathcal{B}_{||.||_U}(0, 1]$.

Aufgabe 2. (i) Let $||.||$ and $||.||'$ be norms on $\mathbb{R}^n$, and let $||.||_*$ and $||.||'_*$ be norms on $\mathbb{R}^m$. If $X \subseteq \mathbb{R}^n$ and $f : X \to \mathbb{R}^m$, then f is Lipschitz with respect to $||.||$ and $||.||_*$ iff f is Lipschitz with respect to $||.||'$ and $||.||'_*$.
(ii) Find the largest $A > 0$ and the smallest $B > 0$ such that for every $x \in \mathbb{R}^n$

$$A|x| \leq |x|_{\text{sum}} \leq B|x|.$$

(iii) Let E be a normed space. If $T : \mathbb{R}^n \to E$ is linear, then f is Lipschitz.

Aufgabe 3. Let $x, y \in \mathbb{R}^n$ such that $|y - x| > 0$.
(i) The function $\gamma_{x,y} : [0, |y - x|] \to \mathbb{R}^n$, defined by

$$\gamma_{x,y}(t) := x + t\frac{y - x}{|y - x|},$$

for every $t \in [0, |y - x|]$ is a C^∞ path from x to y, which is an *isometry* i.e., for every $s, t \in [0, |y - x|]$

$$|\gamma_{x,y}(s) - \gamma_{x,y}(t)| = |s - t|.$$

(ii) If $\delta_{x,y} : [0, |y - x|] \to \mathbb{R}^n$ is a path from x to y that is an isometry, then $\delta_{x,y}$ is equal to $\gamma_{x,y}$.

Aufgabe 4. (i) An inner product $\langle\langle \cdot, \cdot \rangle\rangle$ on $\mathbb{R}^n$ is a continuous function.
(ii) Let I be an interval in $\mathbb{R}$ and let $f, g : I \to \mathbb{R}^n$ be C^1.
(a) If $\langle\langle f, g \rangle\rangle : I \to \mathbb{R}$ is defined for every $t \in I$ by

$$\langle\langle f, g \rangle\rangle(t) := \langle\langle f(t), g(t) \rangle\rangle,$$

then, for every $t \in I$ we have that

$$\langle\langle f, g \rangle\rangle'(t) = \langle\langle f'(t), g(t) \rangle\rangle + \langle\langle f(t), g'(t) \rangle\rangle.$$

(b) For every $t \in I$ we have that

$$\langle\langle f'(t), f(t) \rangle\rangle = \frac{1}{2}\big(||f(t)||^2\big)'.$$

Abgabe. Donnerstag, 26. April 2018 in der Vorlesung.

Besprechung. Montag, 30. April 2018, in der Übung.