LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Dr. Iosif Petrakis
N. Köpp

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Gewöhnliche Differentialgleichungen

Blatt 12 (Probeklausur)

Aufgabe 1. Find the general solution to the following system of odes:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) + x_3(t),\\ \dot{x}_2(t) &= x_2(t) - x_3(t),\\ \dot{x}_3(t) &= x_1(t) + x_3(t).\end{aligned}$$

Aufgabe 2. Find the general solution to the following system of odes:

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) + 2x_3(t),\\ \dot{x}_2(t) &= x_2(t) + x_3(t),\\ \dot{x}_3(t) &= -2x_1(t) + x_3(t).\end{aligned}$$

Aufgabe 3. If $L(\mathbb{R}^n)^{-1}$ is the set of all invertible operators in $L(\mathbb{R}^n)$, and if $T \in L(\mathbb{R}^n)$ such that $||T|| < 1$, we know that
(a) the series $\sum_{k=0}^{\infty} T^k$ converges,
(b) $I_n - T \in \mathrm{L}(\mathbb{R}^\mathrm{n})^{-1}$, and

$$\sum_{k=0}^{\infty} T^k = \frac{1}{I_n - T}.$$

Let the function ${}^{-1} : L(\mathbb{R}^n)^{-1} \to L(\mathbb{R}^n)^{-1}$ be defined by

$$T \mapsto T^{-1}.$$

Show the following:
(i) The function ${}^{-1}$ is continuous at I_n.
(i) The function ${}^{-1}$ is continuous on $L(\mathbb{R}^n)^{-1}$.

Aufgabe 4. Find the general solution of the ode

$$x^{(3)}(t) + 4\ddot{x}(t) + 5\dot{x}(t) + 2x(t) = 0.$$

Aufgabe 5. The Picard method of solving an ode is the method of proof of the fundamental local theorem. Use the Picard method to solve the ode

$$\dot{x} = x + 2 \; ; \quad x(0) = 2,$$

and determine the domain of the solution.

Aufgabe 6. Let $b > 0$ and let $u : [0, b] \to [0, +\infty)$ and $v : [0, b] \to [0, +\infty)$ be continuous functions. If $C > 0$ such that for every $t \in [0, b]$

$$u(t) \leq C + \int_0^t u(s)v(s)ds,$$

and if

$$V(t) = \int_0^t v(s)ds,$$

show that for every $t \in [0, b]$ we have that

$$u(t) \leq Ce^{V(t)}.$$

Besprechung. Dienstag, 10. Juli 2018, in der Vorlesung.