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Sommersemester 18
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Gewöhnliche Differentialgleichungen

Blatt 11

Aufgabe 1. Let $a > 0$ and let $u : [0, a] \to [0, +\infty)$ be continuous. If $L \geq 0$ such that for every $t \in [0, a]$

$$u(t) \leq \int_0^t Lu(s)ds,$$

show that for every $t \in [0, a]$ we have that

$$u(t) = 0.$$

Aufgabe 2. Let $f : \boldsymbol{S} \to \mathbb{R}^n$ be C^1. Show that for every $x_0 \in \boldsymbol{S}$, there is a maximum open interval (α, β), where $\alpha, \beta \in \mathbb{R} \cup \{-\infty, +\infty\}$, such that the following hold:
(i) $0 \in (\alpha, \beta)$, and
(ii) there is a solution $x : (\alpha, \beta) \to \boldsymbol{S}$ of the ode $\dot{x} = f(x)$ such that $x(0) = x_0$.

Aufgabe 3. Let $K \subseteq \boldsymbol{S}$ be compact, $x_0 \in K$, and let $f : \boldsymbol{S} \to \mathbb{R}^n$ be C^1. Suppose that every solution $x : [0, \beta] \to \boldsymbol{S}$ with $x(0) = x_0$ satisfies the property

$$\forall_{t \in [0,\beta]} \big(x(t) \in K\big).$$

Show that there is a solution $x^* : [0, +\infty) \to \boldsymbol{S}$ with $x^*(0) = x_0$ and

$$\forall_{t \geq 0} \big(x^*(t) \in K\big).$$

Aufgabe 4. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be continuous such that $\forall_{x \in \mathbb{R}^n} \big(|f(x)| \leq M\big)$, and let $x_0 \in \mathbb{R}^n$. Moreover, let $(x_n)_{n=0}^{\infty}$ be a sequence of functions from $[0, 1]$ to $\mathbb{R}^n$ such that
(i) x_n is a solution of the ode $\dot{x} = f(x)$, for every $n \in \mathbb{N}$, and
(ii) $\lim_{n \to \infty} x_n(0) = x_0$.
Show that there is a subsequence of $(x_n)_{n=0}^{\infty}$ that converges uniformly on $[0, 1]$ to a solution of $\dot{x} = f(x)$.

Abgabe. Donnerstag, 05. Juli 2018 in der Vorlesung.

Besprechung. Montag, 09. Juli 2018, in der Übung.