LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Gewöhnliche Differentialgleichungen

Blatt 10

Aufgabe 1. (i) Let the space $S(a_1, \ldots, a_n)$ be as in Corollary 1.8.13, and let $s \in S(a_1, \ldots, a_n)$. If $m \geq 1$, then for every $\lambda \in \mathbb{C}$ show that

$$(D - \lambda I_{S(a_1,\ldots,a_n)})^m s = e^{\lambda t} D^m \big(e^{-\lambda t} s\big).$$

(ii) Let S_λ be as in Theorem 1.8.15. Show that the functions

$$e^{\lambda t}, te^{\lambda t}, \ldots, t^{m-1} e^{\lambda t}$$

are linearly independent.

Aufgabe 2. Let the following ode

$$s^{(4)}(t) + 4s^{(3)}(t) + 5\ddot{s}(t) + 4\dot{s}(t) + 4s(t) = 0. \tag{1}$$

(i) Find the general solution of (1).

(ii) Find the solution of (1) that satisfies the following initial conditions:

$$s(0) = 0, \quad \dot{s}(0) = -1, \quad \ddot{s}(0) = -4, \quad s^{(3)}(0) = 14.$$

Aufgabe 3. (i) Give an example of a locally Lipschitz function f that is not Lipschitz.

(You need to explain why f is locally Lipschitz, and you need to show that f is not Lipschitz)

(ii) Prove lemma 2.1.8.

(iii) Show that the function $x : J \to \mathbb{R}^n$ that is defined in the proof of Theorem 2.1.11 as the uniform limit of the functions $(u_n)_{n=0}^{\infty}$, where $u_n : J \to V_{x_0}$, is also a function from J to V_{x_0}.

Aufgabe 4. Let $\boldsymbol{S} = \mathbb{R}$, $x_0 \in \mathbb{R}$, and $f : \boldsymbol{S} \to \mathbb{R}$, defined by $f(x) = x$, for every $x \in \mathbb{R}$. Using the corresponding to f sequence of functions $(u_n)_{n=0}^{\infty}$ defined in the proof of Theorem 2.1.11, show that the solution $x(t)$ of the ode $\dot{x} = x$ in $\mathbb{R}$ that satisfies $x(0) = x_0$ is $x(t) = x_0 e^t$.

Abgabe. Donnerstag, 28. Juni 2018 in der Vorlesung.

Besprechung. Montag, 02. Juli 2018, in der Übung.