LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Sommersemester 18
12.04.2018

Gewöhnliche Differentialgleichungen

Blatt 1

Aufgabe 1. (i) Determine for which $a \in \mathbb{R}$ the function

$$d_a(x, y) := |x - y|^a$$

is a metric on $\mathbb{R}$.

(ii) Let $(X, \langle\langle\cdot,\cdot\rangle\rangle)$ be an inner product space and let $||.||$ be the norm on X induced by $\langle\langle\cdot,\cdot\rangle\rangle$. If $x, y \in X$, the following hold:

(a)

$$|\langle\langle x, y\rangle\rangle| \leq ||x||\ ||y||.$$

(b)

$$|\langle\langle x, y\rangle\rangle| = ||x||\ ||y|| \ \Leftrightarrow\ \langle\langle y, y\rangle\rangle x = \langle\langle x, y\rangle\rangle y.$$

(c)

$$||x + y|| = ||x|| + ||y|| \ \Leftrightarrow\ ||y||x = ||x||y.$$

Aufgabe 2. Complete the proof of Theorem 1.1.6.

Aufgabe 3. Let $(X, ||.||)$ be a normed space.

(i) The norm $||.||$ is a convex function, which is not strictly convex.

(ii) If the norm $||.||$ is induced by some inner product $\langle\langle\cdot,\cdot\rangle\rangle$ on X, then $(X, ||.||)$ is a strictly convex normed space.

(iii) Give an example of norm that is not induced by some inner product.

(iv) If $||.||$ is induced by some inner product, then the function $||.||^2$ is a strictly convex function.

Aufgabe 4. Let $(X, ||.||)$ be a normed space. If $f : X \to \mathbb{R}$ is linear and $f \neq 0$, then f maps open sets of X onto open sets of $\mathbb{R}$.

Abgabe. Donnerstag, 19. April 2018 in der Vorlesung.

Besprechung. Montag, 23. April 2018, in der Übung.