LMU

LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

Prof. Dr. Hans-Dieter Donder
Iosif Petrakis

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Modelle der Mengenlehre

Exam-Results

Exercise 1: **a.** The axiom of foundation is the following formula

$$\forall_x(\exists_y(y \in x) \to \exists_y(y \in x \ \wedge \ \neg\exists_z(z \in x \ \wedge \ z \in y))).$$

❑ True

b. $\mathrm{ZF} \vdash \exists_x(x \in x)$.

❑ False

c. The following axiom of ZF proves the formula

$$\forall_x(x \notin y \ \vee \ y \notin x).$$

❑ Foundation

d. $(\mathrm{Kard} \in V) \quad \vee \quad (\mathrm{On} - \mathrm{Kard} \notin V)$.

❑ True

Exercise 2: The operations below are between ordinals.

a. $\mathrm{rn}(y) < \mathrm{rn}(x) \to y \in x$.

❑ False

b. $\mathrm{rn}(\omega \cdot \omega + \omega) = \mathrm{rn}(\omega \cdot \omega + \omega + 1)$.

❑ False

c. $\omega \cdot \omega + \omega \in V_{\omega \cdot \omega + \omega + 1}$.

❑ True

d. $\{\mathrm{rn}(x) \mid x \in {}^{\mathbb{R}}\mathbb{R}\}$ is bounded in On.

❑ True

Exercise 3: **a.** If $\lambda > \omega$ is a limit ordinal, there exists the immediate previous limit ordinal to λ.

❑ False

b. If $F : \mathrm{On} \to \mathrm{On}$ is increasing i.e., $\forall_{\alpha,\beta\in\mathrm{On}}(\alpha < \beta \to F(\alpha) < F(\beta))$, then

$$\exists_{\alpha\in\mathrm{On}}(F(\alpha) < \alpha).$$

❑ False

c. If $F_1, F_2 : \mathrm{On} \to \mathrm{On}$ such that

$$\{\alpha \in \mathrm{On} \mid F_1(\alpha) = \alpha\} \text{ is a closed and unbounded class,}$$

$$\{\beta \in \mathrm{On} \mid F_2(\beta) = \beta\} \text{ is a closed and unbounded class.}$$

Then the class

$$\{\gamma \in \mathrm{On} \mid F_1(\gamma) = F_2(\gamma) = \gamma\} \text{ is closed and unbounded.}$$

❑ True

d. Please give an example of an unbounded class of ordinals which is not closed.

The successor ordinals

Exercise 4: **a.** The transitive closure of $\{0, 1, \{\omega\}\}$ is

❑ $\omega \cup \{\omega, \{\omega\}\}$

b. If λ is a limit ordinal, then $V_\lambda \models$ Infinity axiom.

❑ False

c. $\mathrm{cf}(\omega_\omega) = \omega$.

❑ True

d. $\omega_3^{\mathrm{cf}(\omega_3)} \leq \omega_3$.

❑ False

Exercise 5: The following relations and formulas are absolute for transitive models of ZF^-:

a. $x \in u \times v$.

❑ True

b. $x \in \mathrm{dom}(r)$.

❑ True

c. α is a limit ordinal.

❑ True

d. $u = \mathcal{P}(v)$.

❑ False

Exercise 6: **a.** $V \neq L$.

❏ Undecidable in ZF

b. HOD is an inner model of ZF.

❏ True in ZF

c. There is no well-ordering on $(\bigcup \mathcal{P}(\omega))^{\mathrm{HOD}}$.

❏ False in ZF

d. $\forall_{n \in \omega}(L_n \subsetneq V_n)$.

❏ False in ZF

Exercise 7: **a.** $\mathrm{Con(ZF)} \to \mathrm{Con}(\mathrm{ZF} + V \neq L)$.

❏ True

b. $V = L \to V = \mathrm{HOD}$.

❏ True

c. $V = L \to \neg\mathrm{GCH}$.

❏ False

d. $\mathrm{Con(ZFC)} \to \mathrm{Con}(\mathrm{ZFC} + V \neq L)$.

❏ True

Exercise 8: **a.** $u \in \mathrm{Def}(u)$.

❏ True

b. $x_1, \ldots, x_n \in u \to \{x_1, \ldots, x_n\} \in \mathrm{Def}(u)$.

❏ True

c. $x, y \in \mathrm{Def}(u) \to x \cup y \notin \mathrm{Def}(u)$.

❏ False

d. u is transitive $\to u \subseteq \mathrm{Def}(u) \ \wedge\ \mathrm{Def}(u)$ is transitive.

❏ True

Exercise 9: Suppose that $<\mathbb{P}, \leq, \mathbb{1}>$ is a set of conditions contained in a countable and transitive model M of ZFC.

a. For every $p \in \mathbb{P}$ and every G generic over M it holds $K_G(\{< \emptyset, p >\}) = \emptyset$.

❑ False

b. Suppose that $p, q \in \mathbb{P}$ such that p, q are incompatible. Then there exists G generic over M such that $p \notin G$.

❑ True

c. If G is $\mathbb{P}$-generic over M and

$$\forall_{p\in\mathbb{P}} \exists_{q_1,q_2\in\mathbb{P}} (q_1 \leq p \ \wedge \ q_2 \leq p \ \wedge \ q_1, q_2 \text{ are incompatible}),$$

then $G \in M$.

❑ False

d. If G is generic over M and

$$\forall_{p,q\in\mathbb{P}} (p \leq q \ \vee \ q \leq p),$$

then $G \notin M$.

❑ False

Exercise 10: Suppose that M is a countable transitive model for ZFC, $\mathbb{P}$ is the set of the finite partial functions from ω to 2 i.e.,

$$\mathbb{P} = \{p \mid p \subset \omega \times 2 \ \wedge \ |p| < \omega \ \wedge \ p \text{ is a function}\},$$

while $p \leq q \leftrightarrow p \supseteq q$. Also, G is $\mathbb{P}$-generic over M and Φ is a name for $\bigcup G$.

a. $\emptyset \Vdash \Phi$ is a function from $\hat{\omega}$ to $\hat{2}$.

❑ True

b. $\emptyset \Vdash \hat{1} \in \mathrm{rng}(\Phi)$.

❑ True

c. $\{< 0, 1 >, < 2, 1 >\} \Vdash \Phi(\hat{1}) \neq \hat{0}$.

❑ False

d. $\{< 0, 0 >, < 10, 1 >, < 11, 1 >\} \Vdash \Phi(\hat{1}) = \hat{0}$.

❑ False