LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Modelle der Mengenlehre

Exam

Family Name:	__________	First name:	__________
Student ID:	__________	Term:	__________
Degree course:	Bachelor, PO ❑ 2007 ❑ 2010 ❑ 2011	Master, PO ❑ 2010 ❑ 2011	
	Lehramt Gymnasium:	❑ modularisiert	❑ nicht modularisiert
	❑ Diplom	❑ Other: __________	
Major subject:	❑ Mathematik ❑ Wirtschaftsm. ❑ Inf. ❑ Phys. ❑ Stat. ❑ ____		
Minor subject:	❑ Mathematik ❑ Wirtschaftsm. ❑ Inf. ❑ Phys. ❑ Stat. ❑ ____		
Credit Points	to be used for ❑ Hauptfach ❑ Nebenfach (Bachelor / Master)		

Please switch off your mobile phone and do not place it on the table; place your identity and student ID cards on the table so that they are clearly visible.
Please check that you have received all **ten problems**.
Please do not write with the colors red or green. Write **on every page** your **family name and your first name**.
Please make sure to submit only one solution for each problem; cross out everything that should not be graded.
If you need a tutorial certificate (Übungsschein / nicht modularisiert) please fill in the form on the second next page.
By entering a pseudonym (e.g. the last four digits of your student ID number) in the appropriate box on the left at the top of the next page you will give your permission to the publication of your results on the lecture's homepage.

You have **120 minutes** in total to complete this examination.

Good luck!

Pseudonym	1	2	3	4	5
	/4	/4	/4	/4	/4

	6	7	8	9	10	$\sum$
	/4	/4	/4	/4	/4	/40

Dieser Leistungsnachweis entspricht auch den Anforderungen

nach §	Abs.	Nr.	Buchstabe	LPO I
nach §	Abs.	Nr.	Buchstabe	LPO I

UNIVERSITÄT MÜNCHEN

ZEUGNIS

Der / Die Studierende der ____________________

Herr / Frau ____________________ *aus* ____________________

geboren am __________ *in* __________ *hat im* __________*–Halbjahr* __________

meine Seminar-Übungen ____________________

____

mit ____________________ *besucht.*

Er / Sie hat ____________________

schriftliche Arbeiten geliefert, die mit ihm / ihr besprochen wurden. __________

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MÜNCHEN, *den* ____________________

Please read carefully each of the following statements, decide whether it is true or false and tick your answer accordingly. In case more than one answers (other than True or False) is given, please tick the one you consider correct.

Each correct answer gives one point. Each false answer gives zero points. The optimal total sum is 40 points.

Exercise 1: **a.** The axiom of foundation is the following formula

$$\forall_x(\exists_y(y \in x) \to \exists_y(y \in x \ \wedge \ \neg\exists_z(z \in x \ \wedge \ z \in y))).$$

❏ True ❏ False

b. $\mathrm{ZF} \vdash \exists_x(x \in x)$.

❏ True ❏ False

c. The following axiom of ZF proves the formula

$$\forall_x(x \notin y \ \vee \ y \notin x).$$

❏ Extensionality ❏ Replacement ❏ Foundation ❏ Union

d. $(\mathrm{Kard} \in V) \quad \vee \quad (\mathrm{On} - \mathrm{Kard} \notin V)$.

❏ True ❏ False

Exercise 2: The operations below are between ordinals.

a. $\mathrm{rn}(y) < \mathrm{rn}(x) \to y \in x$.

❏ True ❏ False

b. $\mathrm{rn}(\omega \cdot \omega + \omega) = \mathrm{rn}(\omega \cdot \omega + \omega + 1)$.

❏ True ❏ False

c. $\omega \cdot \omega + \omega \in V_{\omega \cdot \omega + \omega + 1}$.

❏ True ❏ False

d. $\{\mathrm{rn}(x) \mid x \in {}^{{}^{\mathbb{R}}\mathbb{R}}\mathbb{R}\}$ is bounded in On.

❏ True ❏ False

Exercise 3: **a.** If $\lambda > \omega$ is a limit ordinal, there exists the immediate previous limit ordinal to λ.

❏ True ❏ False

b. If $F : \mathrm{On} \to \mathrm{On}$ is increasing i.e., $\forall_{\alpha,\beta\in\mathrm{On}}(\alpha < \beta \to F(\alpha) < F(\beta))$, then

$$\exists_{\alpha\in\mathrm{On}}(F(\alpha) < \alpha).$$

❏ True ❏ False

c. If $F_1, F_2 : \mathrm{On} \to \mathrm{On}$ such that

$$\{\alpha \in \mathrm{On} \mid F_1(\alpha) = \alpha\} \text{ is a closed and unbounded class,}$$

$$\{\beta \in \mathrm{On} \mid F_2(\beta) = \beta\} \text{ is a closed and unbounded class.}$$

Then the class

$$\{\gamma \in \mathrm{On} \mid F_1(\gamma) = F_2(\gamma) = \gamma\} \text{ is closed and unbounded.}$$

❏ True ❏ False

d. Please give an example of an unbounded class of ordinals which is not closed.

Exercise 4: **a.** The transitive closure of $\{0, 1, \{\omega\}\}$ is

❏ $\{\omega\}$ ❏ $\omega + 1$ ❏ $\omega + 2$ ❏ $\omega \cup \{\omega, \{\omega\}\}$

b. If λ is a limit ordinal, then $V_\lambda \models$ Infinity axiom.

❏ True ❏ False

c. $\mathrm{cf}(\omega_\omega) = \omega$.

❏ True ❏ False

d. $\omega_3^{\mathrm{cf}(\omega_3)} \leq \omega_3$.

❏ True ❏ False

Exercise 5: The following relations and formulas are absolute for transitive models of ZF^-:

a. $x \in u \times v$.

❏ True ❏ False

b. $x \in \mathrm{dom}(r)$.

❏ True ❏ False

c. α is a limit ordinal.

❏ True ❏ False

d. $u = \mathcal{P}(v)$.

❏ True ❏ False

Exercise 6: **a.** $V \neq L$.

❏ True in ZF ❏ False in ZF ❏ Undecidable in ZF

b. HOD is an inner model of ZF.

❏ True in ZF ❏ False in ZF ❏ Undecidable in ZF

c. There is no well-ordering on $(\bigcup \mathcal{P}(\omega))^{\mathrm{HOD}}$.

❏ True in ZF ❏ False in ZF ❏ Undecidable in ZF

d. $\forall_{n \in \omega}(L_n \subsetneq V_n)$.

❏ True in ZF ❏ False in ZF ❏ Undecidable in ZF

Exercise 7: **a.** $\mathrm{Con}(\mathrm{ZF}) \to \mathrm{Con}(\mathrm{ZF} + V \neq L)$.

❏ True ❏ False

b. $V = L \to V = \mathrm{HOD}$.

❏ True ❏ False

c. $V = L \to \neg\mathrm{GCH}$.

❏ True ❏ False

d. $\mathrm{Con}(\mathrm{ZFC}) \to \mathrm{Con}(\mathrm{ZFC} + V \neq L)$.

❏ True ❏ False

Exercise 8: **a.** $u \in \mathrm{Def}(u)$.

❏ True ❏ False

b. $x_1, \ldots, x_n \in u \to \{x_1, \ldots, x_n\} \in \mathrm{Def}(u)$.

❏ True ❏ False

c. $x, y \in \mathrm{Def}(u) \to x \cup y \notin \mathrm{Def}(u)$.

❏ True ❏ False

d. u is transitive $\to u \subseteq \mathrm{Def}(u) \ \wedge\ \mathrm{Def}(u)$ is transitive.

❏ True ❏ False

Exercise 9: Suppose that $<\mathbb{P}, \leq, \mathbb{1}>$ is a set of conditions contained in a countable and transitive model M of ZFC.

a. For every $p \in \mathbb{P}$ and every G generic over M it holds $K_G(\{<\emptyset, p>\}) = \emptyset$.

❏ True ❏ False

b. Suppose that $p, q \in \mathbb{P}$ such that p, q are incompatible. Then there exists G generic over M such that $p \notin G$.

❏ True ❏ False

c. If G is $\mathbb{P}$-generic over M and

$$\forall_{p \in \mathbb{P}} \exists_{q_1, q_2 \in \mathbb{P}} (q_1 \leq p \ \wedge \ q_2 \leq p \ \wedge \ q_1, q_2 \text{ are incompatible}),$$

then $G \in M$.

❏ True ❏ False

d. If G is generic over M and

$$\forall_{p,q \in \mathbb{P}} (p \leq q \ \vee \ q \leq p),$$

then $G \notin M$.

❏ True ❏ False

Exercise 10: Suppose that M is a countable transitive model for ZFC, $\mathbb{P}$ is the set of the finite partial functions from ω to 2 i.e.,

$$\mathbb{P} = \{p \mid p \subset \omega \times 2 \ \wedge \ |p| < \omega \ \wedge \ p \text{ is a function}\},$$

while $p \leq q \leftrightarrow p \supseteq q$. Also, G is $\mathbb{P}$-generic over M and Φ is a name for $\bigcup G$.

a. $\emptyset \Vdash \Phi$ is a function from $\hat{\omega}$ to $\hat{2}$.

❏ True ❏ False

b. $\emptyset \Vdash \hat{1} \in \text{rng}(\Phi)$.

❏ True ❏ False

c. $\{<0, 1>, <2, 1>\} \Vdash \Phi(\hat{1}) \neq \hat{0}$.

❏ True ❏ False

d. $\{<0, 0>, <10, 1>, <11, 1>\} \Vdash \Phi(\hat{1}) = \hat{0}$.

❏ True ❏ False