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Gewöhnliche Differentialgleichungen

Blatt 0

Aufgabe 1. Complete the proof of Theorem 1.1.7.

Aufgabe 2. Picture the unit spheres $\mathcal{S}^1_{|\cdot|}, \mathcal{S}^1_{|\cdot|_{\text{sum}}}$ and $\mathcal{S}^1_{|\cdot|_{\text{max}}}$ of $\mathbb{R}^3$.

Aufgabe 3. Let X be a real vector space and d a metric on X.
(I) Show that the following are equivalent:
(i) There is a norm $||.||$ on X that induces d.
(ii) If $x, y, z \in X$ and $\lambda \in \mathbb{R}$, then d satisfies the following:
(a) $d(x, y) = d(x + z, y + z)$.
(b) $d(\lambda x, \lambda y) = |\lambda| d(x, y)$.
(II) Give an example of a metric on X that has convex ϵ-neighborhoods and it is not induced by some norm on X.

Aufgabe 4. Let $(X, ||.||)$ be a normed space and $A \subseteq X$.
(i) If A is open and $t > 0$, then tA is open.
(ii) If A is a subspace of X, then $A \neq X$ if and only if the interior of A is the empty set.

Besprechung. Montag, 16. April 2018, in der Übung.