Mathematisches Institut
der Universität München
Prof. Dr. Peter Müller

Summer term 2019
Tutorial Sheet 2
30.04.2019

Functional Analysis

T6. Let X be a compact topological space and let Y be a Hausdorff space. Show that every bijective $f \in C(X, Y)$ is a homeomorphism.

T7. Let $J \neq \varnothing$ be an index set. For $j \in J$, let $(X_j, \mathcal{T}_j)$ be a topological space. Let X be the Cartesian product, i.e.

$$X := \bigtimes_{j \in J} X_j := \left\{ x : J \to \bigcup_{j \in J} X_j : x(j) \in X_j \right\}.$$

We define the *product topology* on X as the topology $\mathcal{T}$ given by the base

$$\left\{ \bigtimes_{j \in J} A_j : \forall j \in J : A_j \in \mathcal{T}_j, \text{ with } A_j = X_j \text{ for all but finitely many } j \in J \right\}.$$

Show: $\mathcal{T}$ is the coarsest topology such that all projections $\mathrm{pr}_j : X \to X_j$, $\mathrm{pr}_j(x) := x(j)$, $j \in J$ are continuous.

T8. Let X be a compact topological space. Show that every $f \in C(X, \mathbb{R})$ takes on its maximum and minimum.

If time permits, solve the following supplementary exercise:

T9. Is the set $M := [0, 1] \subseteq \mathbb{R}$ compact with respect to the co-finite topology $\mathcal{T} := \{\varnothing\} \cup \{A \subseteq \mathbb{R} : \mathbb{R} \setminus A \text{ is finite}\}$?