Mathematisches Institut
der Universität München
Prof. Dr. Peter Müller

Summer term 2019
Sheet 2
30.04.2018

# Functional Analysis

**E5** [8 points]. Let $(X, \mathcal{T})$ be a topological space and let $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Show that the set of bounded continuous functions

$$C_b(X) := \Big\{ f : X \to \mathbb{K} \,|\, f \text{ continuous and } \sup_{x \in X} |f(x)| < \infty \Big\}$$

equipped with the metric $d_\infty(f, g) = \sup_{x \in X} |f(x) - g(x)|$ forms a complete metric space.

**E6** [6 points]. Prove that $l^\infty$ is not separable.
*Hint:* Find an uncountable subset $A \subseteq l^\infty$ with the following property: $\forall x \in A \; \exists \, r_x > 0 :$ $B_{r_x}(x) \cap B_{r_y}(y) = \emptyset$ whenever $x \neq y$. Why is this sufficient?

**E7** [6 Points]. Show that compact subsets of Hausdorff spaces are closed.

**E8** [4 Points]. Let $X, Y$ be metric spaces with $X$ compact. Show that every $f \in C(X, Y)$ is *uniformly continuous*, i.e.

$$\forall \, \varepsilon > 0 \, \exists \, \delta \equiv \delta_\varepsilon : \forall \, x \in X : \; f\big(B_\delta(x)\big) \subseteq B_\varepsilon\big(f(x)\big).$$

*Please hand in your solutions until next* **Wednesday (08.05.2018)** *before* **14:00** *in the designated box on the first floor. Don't forget to put your name (max. 2 students per sheet) on all of the sheets you submit.*