MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Dr. Peter Müller

Summer term 2019
Sheet 1
24.04.2019

Functional Analysis

E1 [6 points]. Let $(Y, \mathcal{T})$ be a Hausdorff space.

(i) Prove that the limit of convergent sequences in Y is unique.

(ii) Show that for every $a \in Y$ the singleton $\{a\}$ is closed.

(iii) Let $(X, \mathcal{T}')$ be another topological space, and let $f : X \to Y$ be continuous. Prove that the graph of f, $\Gamma(f) := \{(x, f(x)) \mid x \in X\}$, is closed in $X \times Y$ with respect to the product topology.

E2 [6 points]. Let X be a metric space.
Show Theorem 1.9, i.e. X separable $\Rightarrow$ X 2^{nd} countable.

E3 [6 Points]. Show that the metric space $(C([0,1], \mathbb{R}), d_1)$ defined in Example 1.11 is not complete.

E4 [6 points].

(i) Let (X, d_X) and (Y, d_Y) be complete metric spaces. Let $V \subseteq X$ and $W \subseteq Y$ be dense. Show that every bijective isometry $f : V \to W$ can be uniquely extended to a bijective isometry $\tilde{f} : X \to Y$.

(ii) Show that any two completions of a metric space (X, d) are isometric. (*Note:* This completes the proof of Theorem 1.14.)

Please hand in your solutions until next **Thursday (02.05.2019)** *before* **14:00** *in the designated box on the first floor. Don't forget to put your name (max. 2 students per sheet) on all of the sheets you submit.*