ESTIMATES ON LOWER MOMENTS OF EIGENVALUES VIA KREIN'S SPECTRAL SHIFT FUNCTION

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ABSTRACT. Given two selfadjoint operators $A \geq 0$, $B \geq -c_B$ we extend the estimate

$$\sum_{\lambda \in \sigma(B) \cap (-\infty, 0)} |\lambda|^\gamma \leq C_p(\gamma) \|e^{-B} - e^{-A}\|_{\mathbf{S}_p}^p, \quad \gamma > p,$$

known for $p \geq 1$, to the case of Schatten-classes $\mathbf{S}_p$ with $p < 1$. Furthermore we derive the new sharp result

$$\sum_{\lambda \in \sigma(B) \cap (-\infty, 0)} |\lambda| \leq \|e^{-B} - e^{-A}\|_{\mathbf{S}_1}.$$

The main tool in the derivation of both results is Krein's spectral-shift-function.