Number of eigenvalues and spectral stability due to Schatten norms of semigroup differences

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Abstract: Differences of semigroups play a crucial role in spectral theory. Let A and B be two selfadjoint semibounded operators in $L^2(\Omega)$ generating the semigroups $\{e^{-tA},\ t \geq 0\},\ \{e^{-tB},\ t \geq 0\}$. Then the difference $D_t = e^{-tB} - e^{-tA}$ for some fixed t determine the relations between the spectra of A and B.

The essential spectra are the same if D_t is compact. The absolutely continuous spectra coincide if $e^{-tA}\ D_t\ e^{-tB}$ is a trace class operator. If $A = \sqrt{-\Delta}$, then conditions on D imply the absence of the singularly continuous spectrum.

A new method, developed by G. Katriel, gives estimates for the moments and the number of the negative eigenvalues of B. Using the Jensen identity of complex functions the eigenvalues of B coincide with the zeroes of a constructed holomorphic function. The number of the zeroes are estimated in terms of Hilbert-Schmidt norms or trace class norms of D_t.

For Schrödinger operators the bounds are essentially different to the well-known Lieb-Thirring bounds.