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# **Excercise Sheet 2** for 15. 05. 2017

Let $d \in \mathbb{N}$.

**2.1.** Prove the last statement of Theorem 22 (you may use all the preceding results): For $p \in (1, \infty)$, if a sequence $(f_j)_{j \in \mathbb{N}} \subset L^p$ converges weakly to $f \in L^p$ and $\lim_{j\to\infty} \|f_j\|_p = \|f\|_p$, then $f_j \xrightarrow[j\to\infty]{L^p} f$.

**2.2.** Let $f$ be a non-negative function from $L^1(\mathbb{R}^d)$. Prove that for every $p \in [1, \infty]$ the map $g \mapsto f * g$ is a bounded linear operator in $L^p(\mathbb{R}^d)$ with the norm equal to $\|f\|_1$.

**2.3.** For $p \in [1, \infty)$ let $(f_j)_{j \in \mathbb{N}} \subset L^p(\mathbb{R}^d) \ni f$. Prove or disprove: (a)$\Rightarrow$(b), (b)$\Rightarrow$(a) for

(a) $(f_j)_{j \in \mathbb{N}}$ converges to $f$ in the space of distributions $\mathcal{D}'(\mathbb{R}^d)$.

(b) $(f_j)_{j \in \mathbb{N}}$ converges to $f$ weakly in $L^p(\mathbb{R}^d)$.

**2.4.** Compute the distributional derivative in $\mathcal{D}'(\mathbb{R}^d)$ of the indicator function $\chi_{B_1(0)}$ of the unit ball in $\mathbb{R}^d$. The answer should not contain differentiation.