Mathematisches Institut
LMU München
WS 2013-14

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Mathematical Quantum Mechanics

Homework Sheet 12

Exercise 1: Let $f : \mathbb{R} \to \mathbb{C}$ be a continuous function. For a self–adjoint operator A using the spectral theorem we define the operator $f(A)$.

1. Prove that $\sigma\big(f(A)\big) = \overline{f\big(\sigma(A)\big)}$.
2. Characterize the essential spectrum of $f(A)$ in terms of f and the spectral measure of A.

Exercise 2: Suppose that V is relatively $-\Delta$–compact and E is an isolated eigenvalue of $-\Delta + \lambda_0 V$ for some $\lambda_0 \in \mathbb{R}$.
For $0 < \varepsilon < \mathrm{dist}\big(E, \sigma(-\Delta + \lambda_0 V) \setminus \{E\}\big)$ let

$$P_\lambda := \frac{1}{2\pi \mathrm{i}} \oint_{|z-E|=\varepsilon} (z + \Delta - \lambda V)^{-1} \mathrm{d}z.$$

1. Prove that P_{λ_0} is the projection onto the eigenspace of $-\Delta + \lambda_0 V$ to the eigenvalue E.
2. Prove that P_λ depends analytically on λ in a neighborhood of λ_0.

The solutions should be put to the box marked “Mathematical Quantum Mechanics” on the first floor **by 16:00 on Tuesday, January 21**.