Mathematisches Institut
LMU München
WS 2013-14

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Mathematical Quantum Mechanics

Homework Sheet 11

Exercise 1: Let ρ be the minimizer of the atomic Thomas–Fermi functional. Prove the virial theorem: The total energy E^{TF}, the kinetic energy T, the potential energy $-V$ and the self–repulsion energy D of ρ satisfy

$$-E^{TF} = T = 3D = \frac{3}{7}V.$$

Hint: Rescale ρ and use its minimizing property.

Exercise 2: For the Thomas–Fermi functional of a neutral atom determine the dependence of the ground state energy on the Thomas–Fermi constant γ.

Exercise 3: Let $Z \in \mathbb{N}$. In $\wedge^Z L^2(\mathbb{R}^3 : \mathbb{C}^q)$ set

$$H_Z := \sum_{j=1}^{Z} \big(-\Delta_j - Z|x_j|^{-1} \big).$$

Prove that for large Z the ground state energy of H_Z has the expansion

$$E_Q(Z) = AZ^{7/3} + BZ^2 + O(Z^{5/3})$$

and find the coefficients A and B.

Exercise 4: Consider the Thomas–Fermi functional for a neutral atom of charge Z neglecting the electron–electron repulsion $D(\rho, \rho)$. Determine its minimum and compare the result with that of Exercise 3.

The solutions should be put to the box marked "Mathematical Quantum Mechanics" on the first floor **by 16:00 on Tuesday, January 14**.