Mathematisches Institut der Universität München | Prof. T. Ø. Sørensen, Dr. S. Morozov

SS 2013 | **Advanced Mathematical Quantum Mechanics (MQM2)** | June 19, 2013

Homework 10 for June 26

The following problems are to be handed in (in the designated box near the library on the first floor), at the latest, at 16:00 on June 26.

The numbers of the problems refer to the lecture notes.

Exercise 1: Solve Problem 9.7.

Exercise 2: Let $\mathfrak{H}$ be a separable Hilbert space and $\{f_i\}_{i\in\mathbb{N}}$ an orthonormal basis for $\mathfrak{H}$. Let $|0\rangle$ be the vacuum vector in $\mathcal{F}^B(\mathfrak{H})$. For $M \in \mathbb{N}$ define

$$\Psi_M := \prod_{j=1}^{M} \left[\left(1 - \left(\frac{\nu_j}{\mu_j}\right)^2\right)^{1/4} \sum_{n=0}^{\infty} \left(-\frac{\nu_j}{2\mu_j}\right)^n \frac{a_+^*(f_j)^{2n}}{n!} \right] |0\rangle,$$

where $\mu_j \geqslant 1$ and $\nu_j^2 = \mu_j^2 - 1$ for $j \in \mathbb{N}$. Prove that:

1. $\|\Psi_M\|_{\mathcal{F}^B(\mathfrak{H})} = 1$ for $M \in \mathbb{N}$;
2. for $N > M$,

$$(\Psi_N, \Psi_M)_{\mathcal{F}^B(\mathfrak{H})} = (\Psi_M, \Psi_N)_{\mathcal{F}^B(\mathfrak{H})} = \prod_{j=M+1}^{N} \left(1 - \left(\frac{\nu_j}{\mu_j}\right)^2\right)^{1/4};$$

3. if $\sum_{j=1}^\infty \nu_j^2 < \infty$, then

$$\lim_{M\to\infty} \prod_{j=M+1}^{N} \left(1 - \left(\frac{\nu_j}{\mu_j}\right)^2\right)^{1/4} = 0$$

   uniformly in $N > M$.

Hence, $\{\Psi_M\}_{M\geqslant 1}$ is a Cauchy sequence in $\mathcal{F}^B(\mathfrak{H})$.