Mathematisches Institut der LMU
Prof. Dr. P. Müller
Dr. S. Morozov

Mathematical Quantum Mechanics II
SoSe 2016
16. 06. 2016

Homework 8

For Thursday, 23 June 2016

8.1. Let H be self-adjoint. Prove that:

(a) If $V(H-z)^{-p} \in \mathcal{T}^\infty$ for some $z \in \operatorname{res}(H)$ and $p > 0$, then $V 1_I(H) \in \mathcal{T}^\infty$ for any bounded interval $I \subset \mathbb{R}$.

(b) If V is bounded and $V 1_I(H) \in \mathcal{T}^\infty$ for any bounded interval $I \subset \mathbb{R}$, then $V(H-z)^{-p} \in \mathcal{T}^\infty$ for every $z \in \operatorname{res}(H)$ and $p > 1$.

(c) If V is relatively bounded w.r.t. H with the relative bound zero and $V 1_I(H) \in \mathcal{T}^\infty$ for any bounded interval $I \subset \mathbb{R}$, then $V(H-z)^{-1} \in \mathcal{T}^\infty$ for every $z \in \operatorname{res}(H)$.

(d) $V(H-z)^{-1} \in \mathcal{T}^\infty$ holds for every $z \in \operatorname{res}(H)$ if and only if V is relatively bounded w.r.t. H with the relative bound zero and $V(H-z)^{-p} \in \mathcal{T}^\infty$ for some $p > 0$.

8.2. (a) Let H be self-adjoint in $\mathcal{H}$ and $z \in \operatorname{res}(H)$. Prove that $\lambda \in \sigma_{\text{ess}}(H)$ if and only if there exists a sequence $(x_n)_{n\in\mathbb{N}}$ in $\mathcal{H}$ with

$$x_n \xrightarrow[n\to\infty]{w} 0, \quad x_n \underset{n\to\infty}{\not\to} 0, \quad \text{and} \quad \big((H-z)^{-1} - (\lambda - z)^{-1}\big) x_n \xrightarrow[n\to\infty]{} 0.$$

(b) Let H_0, H be self-adjoint, $z \in \operatorname{res}(H_0) \cap \operatorname{res}(H)$, and $(H_0 - z)^{-1} - (H-z)^{-1} \in \mathcal{T}^\infty$. Prove that $\sigma_{\text{ess}}(H) = \sigma_{\text{ess}}(H_0)$ holds.