

Homework 6

For Thursday, 9 June 2016

6.1. Prove that for $d \leq 3$ the operator $H = -\Delta + V$ with

$$\sup_{x \in \mathbb{Z}^d} \|V\|_{L^2((x-1, x+1)^d)} < \infty$$

is well-defined and self-adjoint in the Hilbert space $L^2(\mathbb{R}^d)$ on the same domain as $-\Delta$.

6.2. Prove that $(\mathcal{T}^2, \|\cdot\|_2)$ is a separable Banach space. Moreover, for any $T_1, T_2 \in \mathcal{T}^2$ we have $T_1 T_2 \in \mathcal{T}^1$ and $\|T_1 T_2\|_1 \leq \|T_1\|_2 \|T_2\|_2$ holds.

6.3. Prove Corollary 4.12: For $d \leq 3$ and $V \in L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ the wave operators $W_{\pm}(-\Delta + V, -\Delta)$ exist and are complete.

6.4. Prove Lemma 4.15.