Mathematisches Institut der LMU
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Mathematical Quantum Mechanics II
SoSe 2016
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Homework 10

For Thursday, 7 July 2016

10.1. Determine for which self-adjoint operators H and $z \in \mathbb{C}$ the integral

$$\int_0^\infty \mathrm{e}^{-t(H-z)} \mathrm{d}t$$

exists and in which sense and find its value.

10.2. Suppose that for a pair of self-adjoint operators H_0, H the wave operators $W_\pm\big(H, H_0, \mathcal{H}_{ac}(H_0)\big)$ exist. Consider the scattering operator

$$S(H_0, H) := W_+\big(H, H_0, \mathcal{H}_{ac}(H_0)\big)^* W_-\big(H, H_0, \mathcal{H}_{ac}(H_0)\big).$$

(a) Prove that $S(H_0, H)$ is unitary provided

$$\operatorname{ran}\Big(W_+\big(H, H_0, \mathcal{H}_{ac}(H_0)\big)\Big) = \operatorname{ran}\Big(W_-\big(H, H_0, \mathcal{H}_{ac}(H_0)\big)\Big)$$

holds.

(b) Prove that $S(H_0, H)$ commutes with arbitrary functions of H_0.