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Mathematical Quantum Mechanics II
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# Homework 1

For Thursday, 21. April 2016

**1.1.** Let $H$ be a self-adjoint operator in a separable Hilbert space $\mathcal{H}$. Prove Lemma 1.6: For every $\psi \in \mathcal{H}_\varkappa$ and $\varphi \in \mathcal{H}$ the complex spectral measure $\mu_{\varphi,\psi} := \langle \varphi, 1_{(\cdot)}(H)\psi \rangle$ is

(a) purely atomic, if $\varkappa = \mathrm{pp}$;

(b) absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$, if $\varkappa = \mathrm{ac}$;

(c) free of atoms, if $\varkappa = \mathrm{c}$;

(d) supported on a Labesgue null set, if $\varkappa = \mathrm{s}$;

(e) singular continuous, if $\varkappa = \mathrm{sc}$.

**1.2.** For every self-adjoint operator $H$ and relatively compact operator $C$ prove

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{e}^{\mathrm{i}tH} C \mathrm{e}^{-\mathrm{i}tH} \psi \, \mathrm{d}t = \sum_{\lambda \in \sigma_p(H)} P_{\{\lambda\}}(H) C P_{\{\lambda\}}(H) \psi, \quad \text{for any } \psi \in \mathfrak{D}(H).$$

Prove that the result holds for any $\psi \in \mathcal{H}$ provided $C$ is bounded.