# Advanced Mathematical Quantum Mechanics – Homework 1

Mathematisches Institut der LMU – SS2009
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**To be discussed on:** 6.05.2009 8,30 – 10 a.m., lecture room B-132 (tutorial session)
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**Exercise 1.1.** Let $f : \mathbb{R}_+ \to \mathbb{R}_+$ be a positive and monotone decreasing function. Let $a_1, \dots, a_N$ be $N$ points in $\mathbb{R}^d$ (where $N$ and $d$ are any positive integers). Fix $p > 0$ and assume that $\int_0^{+\infty} f(x)^p r^{d-1} \mathrm{d}r < \infty$. Prove that

$$\int_{\mathbb{R}^d} \big| \, f\big(|x-a_1|\big) + \cdots + f\big(|x-a_N|\big) \, \big|^p \mathrm{d}x \;\leqslant\; \int_{\mathbb{R}^d} \big| \, N f(|x|) \, \big|^p \mathrm{d}x \, .$$