LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

CAS LMU CENTER FOR ADVANCED STUDIES

Mathematical challenges of zero-range Physics: rigorous results and open problems

international workshop

sponsored by a research-in-residence fellowship
at the Center for Advanced Studies of LMU Munich

26-28 February 2014

Center for Advanced Studies,
Seestraße 13, 80802 Munich

Michele Correggi and Alessandro Michelangeli organisers

General programme

Titles and abstracts

General programme

Wed 26		*15:15 opening* 15:30-16:15 – **A. Posilicano** *Non-linear maximal monotone extensions of symmetric operators* *16:30-17:00 – coffee break* 17:00-17:45 – **A. K. Motovilov** *Three body systems with delta interactions with internal degrees of freedom* 18:00-18:45 – **R. Figari** *Spin-dependent point interactions, used in models of quantum environments*
Thu 27	9:30-10:15 – **F. Werner** *Few-body and many-body physics with zero-range interactions: theory and experiments with ultra-cold atomic gases - Part I* *10:30-11:00 – coffee break* 11:00-11:45 – **F. Werner** *Part II* 12:00-13:00 round table I **A. Teta** moderator *13:00-15:30 – catered lunch + lunch break*	15:30-16:15 – **D. Finco** *Two fermions and a test particle: a detailed analysis* *16:30-17:00 – coffee break* 17:00-17:45 – **S. Albeverio** *Some remarks on spectral properties of Schrödinger operators with (many) concentric δ-shells and related topics* 18:00-18:45 – **J. Behrndt** *Spectral properties of Schrödinger operators with singular interactions on Lipschitz surfaces.* *19:45 – workshop dinner at Weienbauer*
Fri 28	9:30-10:15 – **C. Cacciapuoti** *Effective dynamics for shrinking waveguides* *10:30-11:00 – coffee break* 11:00-11:45 – **D. Noja** *Non-linear Schrödinger equation and point interactions: localised defects and concentrated nonlinearities* 12:00-12:45 – **R. Carlone** *Spectral theory of Gesztesy-Šeba realisations of 1D Dirac operators with point interactions* *13:00-15:15 – catered lunch + lunch break*	15:15-16:45 round table II **S. Albeverio** and **G. Dell'Antonio** moderators *16:45 closing*

Titles and abstracts

S. Albeverio
Some remarks on spectral properties of Schrödinger operators with (many) concentric δ-shells and related topics

The study of spectral properties of Schrödinger operators with potentials concentrated on delta shells is reduced to the study of Bessel operators with point interactions.We present some results and alsomake a few remarks on related topics involving point interactions.

J. Behrndt
Spectral properties of Schrödinger operators with singular interactions on Lipschitz surfaces

In this talk we discuss spectral properties of Schrödinger operators with δ and δ'-interactions supported on hypersurfaces, which separate the Euclidean space into finitely many bounded and unbounded Lipschitz domains. It turns out that the combinatorial properties of the Lipschitz partition and the spectral properties of the corresponding operators are related. As one of the main results we present an operator inequality for the Schrödinger operators with δ and δ' interactions which is based on an optimal colouring and involves the chromatic number of the partition. This inequality implies various relations for the spectra of the Schrödinger operators and it allows to transform known results for Schrödinger operators with delta-interactions to Schrödinger operators with delta'-interactions. The talk is based on joint work with Pavel Exner and Vladimir Lotoreichik.

C. Cacciapuoti
Effective dynamics for shrinking waveguides

Consider a quantum particle constrained to move in a network of thin tubes: a natural idea is to approximate its dynamics with an effective dynamics on a graph. One possible approach to justify this approximation is to consider a sequence of Hamiltonians on shrinking domains, and prove the convergence of such a sequence to an Hamiltonian on the graph. The hard task in this approach is the analysis of the convergence in the proximity of the vertex region. Requiring only self-adjointness of the effective Hamiltonian leaves a number of free parameters for the choice of the coupling conditions in the vertices. The appropriate effective Hamiltonians are the ones for which the coupling can be interpreted in terms of models without free parameters. It turnes out that the limit depends substantially on the network dynamics one works with. For tubes with Neumann boundary, one typically obtains free (or Kirchhoff) coupling conditions at the vertices. The case of Dirichlet (hard-wall) boundary is very different from its Neumann counterpart and more difficult to study. The main source of difficulties is that in this case the spectral properties of the dynamics are substantially affected by geometric perturbations of the network. Focusing on Dirichlet networks, I will analyze the simplest case of a waveguide which shrinks onto a prototypical graph made up of two edges and one vertex. Within this model I will explain how the coupling conditions depend on the spectral properties of the Hamiltonian on the waveguide near the threshold of the continuum spectrum.

R. Carlone
Spectral theory of Gesztesy-Šeba realisations of 1D Dirac operators with point interactions

We introduce the Gesztesy-Šeba realisations D_α and D_β of the 1D Dirac differential expression D with point interactions. They are the relativistic analogue of the corresponding singular Schrödinger operators H_α and H_β with δ and δ´ interactions, respectively. Also, we introduce evolution models in the massless case in order to outline a strategy for the case in which the strength of the point interactions is a given function of the solution itself (non linear evolution problems).

R. Figari
Spin-dependent point interactions, used in models of quantum environments

The thorough examination of the evolution of a microscopic system interacting with a large quantum environment is the first essential step to comprehend the so-called quantum measurement problem inside the framework of quantum mechanics. We introduce a simplified model consisting of a particle interacting, via zero-range forces, with an array of spins (or, what is the same, of model-atoms) and we argue that it is possible to produce some details of the long-term evolution of the whole system, particle+environment, in a non-perturbative way.

D. Finco
Two fermions and a test particle: a detailed analysis.

We discuss the model describing two fermions interacting with a test particle via zero range potentials and we construct a self-adjoint extension using quadratic forms. Stability is proved through a decomposition of the interaction into subspaces of fixed angular momentum and a detailed analysis of the reduced interaction. We briefly discuss the existence of other self-adjoint extensions and the generalisation of these results to the case of N fermions and a test particle.

A. K. Motovilov
Three body systems with delta interactions with internal degrees of freedom

In the first part of the talk I will review old results on three-particle systems whose two-body subsystems have extra channels (like in the quark bag models) coupled with the motion channel via the δ-function. In the second part I will discuss more recent results on resonances and the structure of the three-body T-matrices and scattering matrices in unphysical energy sheets. I will also present some numerical computations that disclose the mechanism of formation of Efimov trimers from resonances when the coupling constant varies.

D. Noja
Non-linear Schrödinger equation and point interactions: localised defects and concentrated nonlinearities.

The nonlinear Schröedinger equation (NLSE) is a model for several physical phenomena, the most known of which are the wave propagation in a nonlinear optical medium and the dynamics of Bose

Einstein condensates. It is also a mathematical paradigm for the interplay between dispersion and nonlinearity in Hamiltonian PDEs. In this talk we will consider some models of NLSE with space dependent coefficients represented by zero range interactions. Zero range coefficients appear in NLSE in two ways. The first one is a point defect acting as an external potential, for example a δ or δ' interaction; so the linear part of the NLSE as a Schröedinger operator turns out to be a point interaction. The second way is a spatially inhomogeneous nonlinearity, ideally concentrated at a point. An introduction to these models and to the more recent results and open problems will be given, with an emphasis on stability properties of standing waves.

A. Posilicano
Non-linear maximal monotone extensions of symmetric operators

Given a linear symmetric operator $S \geq 0$, we explicitly define and provide its nonlinear resolvents, nonlinear maximal monotone operators A_Θ (i.e., the generators of one-parameter continuous nonlinear semi-groups of contractions) which coincide with the Friedrichs extension of S on a convex set containing $D(S)$. The extension parameter Θ in ($\boldsymbol{h} \times \boldsymbol{h}$) ranges over the set of nonlinear maximal monotone relations on an auxiliary Hilbert space $\boldsymbol{h}$ isomorphic to the deficiency subspace of S. Moreover, A_Θ is a sub-potential operator (i.e., it is the sub-differential of a lower semicontinuos convex function) whenever Θ is sub-potential. Some examples providing nonlinear singular perturbation of differential operators are presented.

F. Werner
Few-body and many-body physics with zero-range interactions: theory and experiments with ultra-cold atomic gases

Quantum physics with zero-range interactions has a long history, initially motivated by nuclear physics. In the last 15 years, there has been strong revival of interest due to a new generation of experiments on ultra-cold atomic gases near Feshbach resonances. I will give an introductory presentation on such experiments and corresponding theory. In particular, we will see that there are radical differences between the bosonic and fermionic cases (both from a fundamental point of view and from a practical experimental point of view). I will also present recent work on the unitary Fermi gas [1,2] and on the unitary Bose gas [3,4]. I will emphasise some open mathematical challenges, the main one being the very existence of the unitary Fermi gas.

[1] K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov, M. J. H. Ku, A. T. Sommer, L. W. Cheuk, A. Schirotzek, M. W. Zwierlein, "Feynman diagrams versus Fermi-gas Feynman emulator", Nature Phys. 8, 366 (2012)

[2] K. Van Houcke, F. Werner, N. Prokof'ev, B. Svistunov, "Bold diagrammatic Monte Carlo for the resonant Fermi gas", arXiv:1305.3901

[3] B. S. Rem, A. T. Grier, I. Ferrier-Barbut, U. Eismann, T. Langen, N. Navon, L. Khaykovich, F. Werner, D. S. Petrov, F. Chevy, C. Salomon, "Lifetime of the Bose Gas with Resonant Interactions", Phys. Rev. Lett. 110, 163202 (2013)

[4] Y. Castin, F. Werner, "Third virial coefficient of the unitary Bose gas", Canadian Journal of Physics 91 (2013) 382