Mathematical Quantum Mechanics, 2014/15
Homework Problems, LMU

Issued: January 20, 2015; deadline for handing in the solutions: Tuesday, January 27, 2015, 4 pm

38. Prove that for the sequence $\{\psi_n\}$ with $\psi_n(x_1, x_2) = \phi(x_1)\chi_n(x_2)$ constructed in the lecture for the proof of the HVZ theorem, the norm of the remaining potential terms converges to zero, i.e., as $n \to \infty$

$$\int |\frac{1}{|x_1 - x_2|}\psi_n(x_1, x_2) - \frac{Z}{|x_2|}\psi_n(x_1, x_2)|^2 dx_1 dx_2 \to 0. \tag{44}$$

39. Consider a finite dimensional matrix $A = ((a_{ij}))$ in Jordan normal form with an eigenvalue λ_0, and a small enough $\varepsilon > 0$ such that λ_0 is the only eigenvalue of A within a disc of radius ε around λ_0. Show that by

$$P = -\frac{1}{2\pi \mathrm{i}} \oint_{|\lambda - \lambda_0| = \varepsilon} (A - \lambda)^{-1} d\lambda \tag{45}$$

a projection is defined that projects onto the set $\{v \mid \exists n > 0 : (A - \lambda_0)^n v = 0\}$. Hint: Set $a_{ij} = (\mu - \lambda)\delta_{i,j} + \delta_{i+1,j}$ and prove that

$$\left(A^{-1}\right)_{ij} = \delta_{i,j}(\mu - \lambda)^{-1} - \delta_{i+1,j}(\mu - \lambda)^{-2} + \delta_{i+2,j}(\mu - \lambda)^{-3} + \dots. \tag{46}$$