

Mathematical Quantum Mechanics, 2014/15

Homework Problems, LMU

Issued: December 16, 2014; extended deadline for handing in the solutions: Thursday, January 8, 2015, 4 pm

31. Consider a potential $V \in C_0^\infty(\mathbb{R}^\nu)$ with $\nu \geq 3$ and $V \geq 0$ and the eigenstates and eigenvalues

$$(-\hbar^2\Delta - V)\psi_n = \lambda_n\psi_n \quad (35)$$

of the corresponding Schrödinger operator. Show that as $\hbar \downarrow 0$ the sum of the eigenvalues is approximated by the classical phase space integral, viz.,

$$\sum_{\lambda_n < 0} \lambda_n = (2\pi\hbar)^{-\nu} \left(\int d^\nu p d^\nu q (p^2 - V(q))_- + O(\hbar) \right). \quad (36)$$