LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Winter term 2021

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Mathematical Gauge Theory II

Sheet 13

Exercise 1. (Spheres with self-intersection zero) Suppose that Y is a smooth closed oriented 4-manifold with $b_2^+(Y) \geq 2$ which contains a smoothly embedded $S^2 \hookrightarrow Y$ of self-intersection zero, representing a class of infinite order in $H_2(Y;\mathbb{Z})$. Consider the manifold $X = Y\#\overline{\mathbb{CP}}^2$.

1. Prove that there exist infinitely many pairwise distinct classes $S_i \in H_2(X;\mathbb{Z})$ represented by embedded 2-spheres of self-intersection -1.
2. Show that if there exists a Spin^c-structure $\mathfrak{s}$ on X with non-zero Seiberg-Witten invariant, then there are infinitely many such structures. Conclude that the Seiberg-Witten invariants of X and Y are in fact identically zero.

Note: You may use the following fact: If Y has $b_2^+(Y) \geq 2$ and $\mathfrak{s} \in \mathrm{Spin}^c(Y)$, then there is a $\mathfrak{s}' \in \mathrm{Spin}^c(Y\#\overline{\mathbb{CP}}^2)$ with $SW_Y(\mathfrak{s}) = SW_{Y\#\overline{\mathbb{CP}}^2}(\mathfrak{s}')$.

Exercise 2. (Seiberg-Witten invariants of $p\mathbb{CP}^2\#q\overline{\mathbb{CP}}^2$) Let $X = p\mathbb{CP}^2\#q\overline{\mathbb{CP}}^2$. Suppose that $p, q \geq 2$. Prove that $SW_X \equiv 0$.

You can hand in solutions in the lecture on Thursday, 17 February 2022.