Winter term 2021

Prof. D. Kotschick
S. Gritschacher

Mathematical Gauge Theory II

Sheet 3

Exercise 1. (Associated $\mathrm{Spin}^c(n)$-bundles) Let $Q \longrightarrow X$ be a principal $\mathrm{Spin}^c(n)$-bundle with an isomorphism $Q/S^1 \cong \mathrm{Fr}(H)$ for an oriented Euclidean vector bundle $H \longrightarrow X$. Consider the vector bundle $V \longrightarrow X$ associated to Q by the standard representation

$$\begin{aligned} \mathrm{Spin}^c(n) &\longrightarrow \mathrm{U}(N) \\ (\tau, \sigma) &\mapsto \quad \sigma\ . \end{aligned}$$

Show that the standard Clifford module γ_0 induces a Clifford module γ for V.

Exercise 2. (Twisting of Spin^c-structures I) Let $(V_{\mathfrak{s}}, \gamma_{\mathfrak{s}})$ be a Spin^c-structure and L a complex line bundle. Show that the pair $(V_{\mathfrak{s}'}, \gamma_{\mathfrak{s}'})$ has a Clifford module structure, where $V_{\mathfrak{s}'} := V_{\mathfrak{s}} \otimes L_\delta$, $i\colon \mathrm{End}(V_{\mathfrak{s}}) \longrightarrow \mathrm{End}(V_{\mathfrak{s}'})$ is an isomorphism, and $\gamma_{\mathfrak{s}'} = i \circ \gamma_{\mathfrak{s}}$.

Exercise 3. (Twisting of Spin^c-structures II) Let $\mathfrak{s}$ be a Spin^c-structure and L a complex line bundle. Show that for the Spin^c-structure $\mathfrak{s}' := \mathfrak{s} \otimes L$ the characteristic line bundle satisfies

$$L_{\mathfrak{s}'} = L_{\mathfrak{s}} \otimes L^2,$$

where $L^2 = L \otimes L$.

Exercise 4. (Intersection form of connected sums) Let M_1 and M_2 be closed, oriented, connected 4-manifolds with intersection forms Q_{M_1} and Q_{M_2} respectively. Show that the intersection form of their connected sum $M_1 \# M_2$ is given by

$$Q_{M_1 \# M_2} = Q_{M_1} \oplus Q_{M_2}\ .$$

You can hand in solutions in the lecture on Thursday, 11 November 2021.