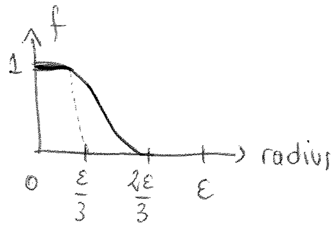
Ex. 1. Here are some brief notes for the 2-dim case

(this particular exercise won't be relevant for the exam, but if you still want to discuss it, feel free to come to my office).

Since the Künneth structure on M is flat, it is locally isomorphic to the standard structure $(\omega_0, \mathcal{F}_0, \mathcal{G}_0)$ on $\mathbb{R}^2$, i.e., $\omega_0 = dx \wedge dy$ and $\mathcal{F}_0$ (resp. $\mathcal{G}_0$) has leaves $\mathbb{R} \times \{c\}$ (resp. $\{c\} \times \mathbb{R}$) for $c \in \mathbb{R}$.

This means we can find a Darboux chart around $p \in M$, i.e. a diffeomorphism $\varphi : \underset{p \mapsto 0}{\mathcal{U} \to B_\varepsilon(0)} \subseteq \mathbb{R}^2$, $\varepsilon > 0$ where $\mathcal{U}$ is an open nbhd of p, which takes $\mathcal{F}$ to $\mathcal{F}_0$ and $\mathcal{G}$ to $\mathcal{G}_0$ and satisfies $\varphi^*(dx \wedge dy) = \omega|_{\mathcal{U}}$.

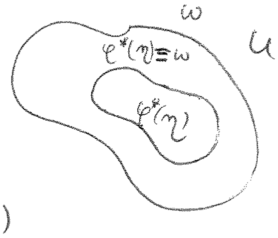
Now we modify the non-flat Künneth structure from Sheet 9 Ex. 4 to make it standard outside a small nbhd of $(0,0) \in \mathbb{R}^2$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a smooth cutoff function of the form



, i.e., which is 1 in $B_{\varepsilon/3}(0)$ and 0 outside $\overline{B_{2\varepsilon/3}(0)}$.

Define $\eta := \frac{1}{2}\left(2 + f(x,y) \sin(2\pi x) \sin(2\pi y)\right) dx \wedge dy$. Then $(\eta, \mathcal{F}_0, \mathcal{G}_0)$ is a non-flat Künneth structure which agrees with $(\omega_0, \mathcal{F}_0, \mathcal{G}_0)$ outside $\overline{B_{2\varepsilon/3}(0)}$.

→ since by our computation on Sheet 9 it was nonflat in the origin



By construction, $\varphi^*(\eta)|_{\mathcal{U} \setminus \varphi^{-1}(\overline{B_{2\varepsilon/3}(0)})} = \omega|_{\mathcal{U} \setminus \varphi^{-1}(\overline{B_{2\varepsilon/3}(0)})}$

and so we can glue the two symplectic forms to obtain a modified symplectic form ω' on M. This ω' is diffeomorphic to η in a small nbhd $\varphi^{-1}(B_{\varepsilon/3}(0)) \subseteq U$ of p, and agrees with ω outside U. Moreover, $\mathcal{F}$ and $\mathcal{G}$ are still complementary Lagrangian foliations for ω'. So $(\omega', \mathcal{F}, \mathcal{G})$ is ~~the~~ ~~desired~~ a modified Künneth structure on M which is no longer flat at p, as desired.