Summer term 2022

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Symplectic Geometry

Sheet 4

Exercise 1. Let M be a smooth manifold and $\pi\colon T^*M \to M$ its cotangent bundle. Equip T^*M with the canonical symplectic structure $\omega_{\mathrm{can}} = -d\lambda$, where $\lambda \in \Omega^1(T^*M)$ is the Liouville 1-form. Let $\varphi\colon M \to M$ be a diffeomorphism. Show that the diffeomorphism $\psi\colon T^*M \to T^*M$ induced by φ satisfies $\psi^*(\lambda) = \lambda$, and conclude that ψ is a symplectomorphism.

Exercise 2. Let W be a $2n$-dimensional manifold, $Q \subset W$ a compact submanifold and $\omega_0, \omega_1 \in \Omega^2(W)$ closed 2-forms satisfying the assumptions of Moser's theorem. Show that in the proof of Moser's theorem the 1-form σ satisfying $(\omega_0 - \omega_1)|_V = d\sigma$ on a suitable open neighbourhood V of Q can be chosen so that $\sigma|_Q \equiv 0$.

Exercise 3. Find two non-degenerate 2-forms $\omega_0, \omega_1 \in \Omega^2(U)$ on some open subset $U \subset \mathbb{R}^6$ neither of which is closed and which are not diffeomorphic.[1]

Exercise 4. Give a direct proof of the Darboux theorem in the case of a surface Σ by using the fact that every non-vanishing 1-form on Σ can be written locally as $f dg$ for suitable functions f and g.

Please hand in your solutions in the lecture on Friday, 27 May 2022.

[1] By Darboux's theorem any two symplectic forms are locally diffeomorphic, and clearly two 2-forms one of which is closed and one of which is not closed can never be diffeomorphic.