Mathematisches Institut
der Universität München
Prof. Dr. Peter Müller

Summer term 2014
Sheet 13
02.06.2014

Functional Analysis

E50 [7 points]. Let X be a Banach space, $f \in X^*$ and let $(f_n)_{n\in\mathbb{N}}$ be a sequence in X^*. Prove the following statements:

(*i*) If $f_n \xrightarrow{w^*} f$, then $\|f\|_* \leqslant \liminf_{n\to\infty} \|f_n\|_* \leqslant \sup_{n\in\mathbb{N}} \|f_n\|_* < \infty$.

(*ii*) $f_n \xrightarrow{w^*} f$, if and only if

(*a*) $\sup_{n\in\mathbb{N}} \|f_n\|_* < \infty$

(*b*) $\exists A \subset X$ s.th. $\mathrm{span}(A) \subset X$ is dense and $\lim_{n\to\infty} f_n(x) = f(x)$ for all $x \in A$.

E51 [7 points]. For $n \in \mathbb{N}$ let $T_n : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ be the operator of multiplication by $1_{[-n,n]}$, i.e. $T_n f(x) = 1_{[-n,n]}(x) f(x)$ for all $x \in \mathbb{R}$.

(*i*) Show that T_n is bounded and calculate its norm.

(*ii*) Prove or disprove: $(T_n)_{n\in\mathbb{N}}$ converges (*a*) weakly, (*b*) strongly, or (*c*) in norm.

E52 [5 points]. Let $\mathcal{H}$ be a Hilbert space, and $(T_n)_{n\in\mathbb{N}}$ a sequence in $\mathrm{BL}(\mathcal{H})$ such that $(\langle y, T_n x\rangle)_{n\in\mathbb{N}}$ is a Cauchy sequence in $\mathbb{K}$ for all $x, y \in \mathcal{H}$. Prove that there exists $T \in \mathrm{BL}(\mathcal{H})$ such that $T_n \xrightarrow{w} T$ as $n \to \infty$.

E53 [5 points]. Let $\mathcal{H}$ be a Hilbert space and $S, T \in \mathrm{BL}(\mathcal{H})$. Prove that $(TS)^* = S^*T^*$ and $(T^*)^* = T$.

Please hand in your solutions until next **Wednesday (09.07.2014)** *before* **12:00** *in the designated box on the first floor. Don't forget to put your name and the letter of your exercise group on all of the sheets you submit.*

For more details please visit http://www.math.lmu.de/~gottwald/14FA/