MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Dr. Peter Müller

Summer term 2014
Sheet 2
16.04.2014

Functional Analysis

E5 [8 points].

(*i*) Let (X, d_X) and (Y, d_Y) be complete metric spaces. Let $V \subseteq X$ and $W \subseteq Y$ be dense. Show that every bijective isometry $f : V \to W$ can be uniquely extended to a bijective isometry $\tilde{f} : X \to Y$.

(*ii*) Show that any two completions of a metric space (X, d) are isometric. (*Note:* This completes the proof of Theorem 1.24.)

E6 [6 points]. Let $(X, \mathcal{T})$ be a topological space and let $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Show that the set of bounded continuous functions

$$C_b(X) := \Big\{ f : X \to \mathbb{K} \,|\, f \text{ continuous and } \sup_{x \in X} |f(x)| < \infty \Big\}$$

equipped with the metric $d_\infty(f, g) = \sup_{x \in X} |f(x) - g(x)|$ forms a complete metric space.

E7 [6 points]. Prove that l^∞ is not separable. (*Hint:* Find an uncountable subset $A \subseteq l^\infty$ with the following property: $\forall x \in A \; \exists \, r_x > 0 : B_{r_x}(x) \cap B_{r_y}(y) = \emptyset$ whenever $x \neq y$. Why is this sufficient?)

E8 [4 points]. Show that compact subsets of Hausdorff spaces are closed.

Please hand in your solutions until next **Wednesday (23.04.2014)** *before* **12:00** *in the designated box on the first floor. Don't forget to put your name and the letter of your exercise group on all of the sheets you submit.*

For more details please visit http://www.math.lmu.de/~gottwald/14FA/