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Mathematical Quantum Mechanics

Problem Sheet 12

Hand-in deadline: 24.01.2017 before 12:00 in the designated MQM box (1st floor, next to the library).

Exercise 1: Let $(A_k)_{k\in\mathbb{N}} \subset BL(\mathcal{H})$, and assume that A_k converge strongly to $A \in BL(\mathcal{H})$ as $k \to \infty$. Prove: If K is a compact operator on $\mathcal{H}$, then $\lim_{k\to\infty} \|A_k K - AK\| = 0$.

Exercise 2: Prove that the following limits hold strongly as $t \to \infty$:

1. $\mathrm{e}^{-\mathrm{i}tH} W_+ - \mathrm{e}^{-\mathrm{i}tH_0} P_{M_0} \to 0$;
2. $\mathrm{e}^{\mathrm{i}tH_0}\mathrm{e}^{-\mathrm{i}tH} P_M \to W_+^*$;
3. $(W_+^* - \mathbb{I})\mathrm{e}^{-\mathrm{i}tH_0} P_{M_0} \to 0$;
4. $\mathrm{e}^{\mathrm{i}tH_0} W_+^* \mathrm{e}^{-\mathrm{i}tH_0} P_{M_0} \to P_{M_0}$;

Here, the generalized wave operators $W_\pm := W_\pm(H, H_0; M_0)$ are defined for a given closed H_0-reducing subspace $M_0 \subset \mathcal{D}(\Omega_{\pm(H,H_0)})$ by

$$W_\pm(H, H_0; M_0) := \mathrm{s} - \lim_{\mathrm{t}\to\pm\infty} \mathrm{e}^{\mathrm{itH}}\mathrm{e}^{-\mathrm{itH_0}}\mathrm{P}_{\mathrm{M_0}}.$$

Moreover, $M := \operatorname{Ran} W_\pm(H, H_0; M_0) = \Omega_{\pm(H,H_0)})M_0$, and P_{M_0}, P_M are the orthogonal projections onto M_0, M, repsectively.

Exercise 3: Let H_0, H, H' be self-adjoint operators, and let $M_0 \subset \mathcal{D}(\Omega_+(H, H_0))$ and $M \subset \mathcal{D}(\Omega_+(H', H))$ be closed subspaces that reduce H_0 and H, respectively. Prove: If

$$\operatorname{Ran}(W_+(H, H_0; M_0)) = \Omega_+(H, H_0)M_0 \subset M,$$

then $M_0 \subset \mathcal{D}(\Omega_+(H', H_0))$ and

$$W_+(H', H_0; M_0) = W_+(H', H; M)W_+(H, H_0; M_0).$$