Mathematisches Institut
LMU München
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Prof. Dr. P. Müller
Dr. R. Helling
Dr. J.-C. Cuenin

Mathematical Quantum Mechanics

Problem Sheet 1

Hand-in deadline: 25.10.2017 before 12:00 in the designated MQM box (1st floor, next to the library).

Exercise 1: In the physical literature some of the features of quantum theory are often considered to have non-important technical nature. In this exercise we would like to discuss their necessity.

1. Assume that the Born–Jordan commutation relation
$$[p, q] = -\mathrm{i}\hbar \tag{1}$$
is satisfied by $n \times n$ matrices p and q ($n \in \mathbb{N}$) with reduced Planck constant $\hbar$. Conclude that the Planck constant must be zero, i.e., there is no quantum mechanics. Find a proof which does not use eigenvectors.

2. Assume that two observables p and q are defined everywhere on a Hilbert space $\mathcal{H}$, satisfy the Born-Jordan commutation relation (1), and that one of them has an eigenvector. Show that there is no quantum mechanics in the above sense.

3. Still more generally, assume that p and q are defined on a common dense subspace $\mathcal{D} \subset \mathcal{H}$ that is invariant under p and q, i.e. $p\mathcal{D} \subset \mathcal{D}$, $q\mathcal{D} \subset \mathcal{D}$. Prove that, if (1) holds, then at least one of the operators p or q must be unbounded.

Jean-Claude Cuenin