Mathematisches Institut
LMU München
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Prof. H. Siedentop
Dr. R. Helling
Dr. J.-C. Cuenin

Mathematical Quantum Mechanics

Problem Sheet 5

Hand-in deadline: 11/24/2016 before noon in the designated MQM box (1st floor, next to the library).

Exercise 1: The following fact was used in the lecture: If $\{\rho_n\}_{n\in\mathbb{N}}$ converges weakly to ρ in $L^{5/3}(\mathbb{R}^3)$ and also converges weakly to $\widetilde{\rho}$ with respect to the Coulomb scalar product, then $\rho = \widetilde{\rho}$. Prove this.

Exercise 2: *Newton's theorem:* The potential of a charge distribution $\rho : \mathbb{R}^3 \to \mathbb{R}_+$ is given by

$$V(x) = \int_{\mathbb{R}^3} \frac{\rho(y)}{|x-y|} dy.$$

(i) Assuming that ρ is spherically symmetric, prove that V is also spherically symmetric and has the form

$$V(x) = v(r) = \int_{|y|>r} \frac{\rho(y)}{|y|} dy + \frac{1}{r} \int_{|y|<r} \rho(y) dy,$$

where $r = |x|$.

(ii) Determine V for the particular charge distribution

$$\rho(x) = \begin{cases} 4\pi r_0^3/3 & \text{if } |x| < r_0 \\ 0 & \text{otherwise} \end{cases}$$

with $r_0 > 0$.

Exercise 3: The following statement was given in the lecture: For densities ρ and g discussed there, the identity

$$\frac{d}{dt} \int_{\mathbb{R}^3} (\rho(x) + tg(x))^{5/3} dx = \int_{\mathbb{R}^3} \frac{d}{dt} (\rho(x) + tg(x))^{5/3} dx$$

holds at $t = 0$. Prove this statement in detail.