Mathematisches Institut
LMU München
WS 2016-17

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Mathematical Quantum Mechanics

Problem Sheet 11

Hand-in deadline: 01/19/2017 before noon in the designated MQM box (1st floor, next to the library).

To register for the final exam send an email to cuenin@math.lmu.de until 01/16, 8pm!

Ex. 1: Consider the the Hamiltonian $H = \sqrt{1+|p|^2} - 1 - Z/|x|$ on $L^2(\mathbb{R}^3)$, defined in the quadratic form sense for $0 < Z \leq 2/\pi$. Prove that

$$\sigma_{\mathrm{ess}}(H) = [0, \infty) \quad \text{for } Z < 2/\pi.$$

Ex. 2: Let V be a locally bounded positive function with $V(x) \to \infty$ as $|x| \to \infty$.

a) Prove that $-\Delta + V$, defined as a sum of quadratic forms, is self-adjoint.

b) Prove that $-\Delta + V$ has purely discrete spectrum.

Ex. 3: Let H_0 be a nonnegative selfadjoint operator in a Hilbert space $\mathfrak{h}$, and let V be bounded and symmetric. Define $H = H_0 + V$ and let $E < 0$. Prove that the following are equivalent:

i) $E \in \sigma(H)$;

ii) $-1 \in \sigma((H_0 - E)^{-1/2} V (H_0 - E)^{-1/2})$;

iii) $-1 \in \sigma(\mathrm{sgn}(V) V^{1/2} (H_0 - E)^{-1} |V|^{1/2})$.

Can you generalize the statements *i)* $\iff$ *ii)* or *i)* $\iff$ *iii)* to certain unbounded potentials?