Bishop's constructivism in foundations and practice of mathematics

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Aim of this talk

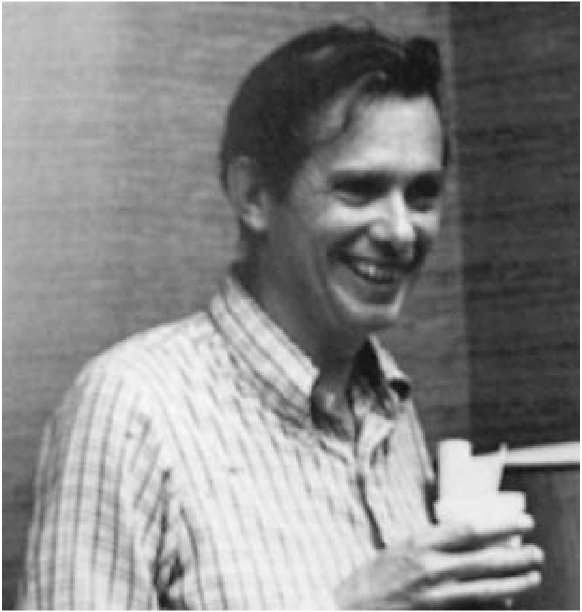
To present some fundamental features of Bishop-style constructive mathematics (BCM) and explain its role in the foundations of mathematics.

Plan of this talk

1. Bishop and his foundational plan.
2. The influence of BCM to the foundations of mathematics.
3. How Bishop succeeded not to contradict with classical mathematics.
4. The fundamental thesis of constructivism.
5. The real numbers under the fundamental thesis.
6. Bishop spaces.
7. The future of BCM.

Bishop and his foundational plan

Errett Bishop (1928-1983)



Bishop's contributions

1. On **polynomial and rational approximation theiry** (extensions of Mergelyan's approximation theorem and the theorem of F. Riesz and M. Riesz concerning measures on the unit circle orthogonal to polynomials.)
2. On the **general theory of function algebras** (Bishop-DeLeeuw theorem, existence of Jensen measures).
3. On **Banach spaces and operator theory** (the Bishop condition, the Bishop-Phelps theorem).
4. On the **theory of functions of several complex variables** (biholomorphic embedding theorem for a Stein manifold, new proof of Remmert's proper mapping theorem).
5. On **constructive mathematics** (basic real and complex analysis, functional analysis, integration and measure theory, theory of Banach algebras).

Halmos, 1985

> *He discovered many of the fundamental concepts about function algebras and the relations among those concepts. Then, almost* **discontinuously**, *he got religion, went into constructive mathematics, wrote the book that made the phrase famous, and started the* **sect** *of which he was the leading but somewhat reluctant guru till the day he died. Functional analysis misses him, and so does constructive mathematics, and so, most of all, do we, his friends.*

Bishop, as Brouwer, was thinking in constructive terms since he was very young.

Bishop wrote the book that made the phrase famous to many classical mathematicians.

E. Bishop: *Foundations of Constructive Analysis*, McGraw-Hill, 1967. (**BISH***)

E. Bishop and D. S. Bridges: *Constructive Analysis*, Grundlehren der Math. Wissenschaften 279, Springer-Verlag, Heidelberg-Berlin-New York, 1985. (**BISH**)

Bishop's foundational program of mathematics

1. To provide a simple **informal** framework for constructive mathematics that looks very similar to classical mathematics, and does not contradict classical mathematics as Brouwer's intuitionistic mathematics. This framework is called BISH.
2. To develop a **formalization** of BISH, after developing large parts of mathematics in BISH.
3. To **implement** the formalization of BISH into some computer language.

Bishop primary aim was to influence the standard mathematician. Instead his work had an enormous impact on the foundations of mathematics and logic.

Informal constructive mathematics is concerned with the communication of algorithms, with enough precision to be intelligible to the mathematical community at large.

Formal constructive mathematics is concerned with the communication of algorithms with enough precision to be intelligible to machines.

The informal system BISH I

Use of intuitionistic logic.

1. The concept of function is primitive, therefore it is not a set.
2. There exists a primitive set of natural numbers.
3. A set X is completely defined when a method to construct an abstract element of X, a method to prove that two elements of X are equal, and a proof that this equality $=_X$ on X is an equivalence relation are given.
4. There is no notion of equality between elements of sets X and Y which are not subsets of some set Z.
5. An *operation*, or a rule, or an algorithm, is a primitive notion. A *function* from a set X to a set Y is an *extensional operation* i.e., $\forall_{x \in X}(f(x) \in Y)$ and $\forall_{x,x' \in X}(x =_X x' \to f(x) =_Y f(x'))$.

The informal system BISH II

1. A subset Y of X is a set for which we can show that $\forall_{y \in Y}(y \in X)$.
2. If X, Y are sets the set $\mathbb{F}(X, Y)$ of all functions from X to Y is formed, where $f =_{\mathbb{F}(X,Y)} g \leftrightarrow \forall_{x \in X}(f(x) =_Y g(x))$, for every $f, g \in \mathbb{F}(X, Y)$. The method of constructing an element of $\mathbb{F}(X, Y)$ is considered to be a proof that $\forall_{x \in X}(f(x) \in Y)$.
3. If B is a rule which associates to every element x of a set A a set $B(x)$, the *sum set*, or *disjoint union* $\sum_{x \in A} B(x)$ and the *infinite product* $\prod_{x \in A} B(x)$ are defined by

$$\sum_{x \in A} B(x) := \{(x, y) \mid x \in A \wedge y \in B(x)\},$$

$$\prod_{x \in A} B(x) := \{f \in \mathbb{F}(A, \bigcup_{x \in A} B(x)) \mid \forall_{x \in A}(f(x) \in B(x))\},$$

where the *exterior union* $\bigcup_{x \in A} B(x)$ is defined by Richman.

The choice principles in BISH

The **principle of dependent choice**, which implies the **principle of countable choice**, and **Myhill's axiom of nonchoice**

$$Q \subseteq X \times X \to x_0 \in X \to \forall_{x \in X} \exists_{y \in X} (Q(x, y)) \to$$

$$\to \exists_{f \in \mathbb{F}(\mathbb{N}, X)} (f(0) = x_0 \wedge \forall_{n \in \mathbb{N}} (Q(f(n), f(n+1)))).$$

$$\forall_{n \in \mathbb{N}} \exists_{x \in X} (P(n, x)) \to \exists_{f \in \mathbb{F}(\mathbb{N}, X)} (\forall_{n \in \mathbb{N}} (P(n, f(n))),$$

$$\forall_{x \in X} \exists !_{y \in Y} (A(x, y)) \to \exists_{f : X \to Y} \forall_{x \in X} (A(x, f(x))).$$

The informal system BISH without countable choice is called **RICH**, as F. Richman advocated the development of constructive math in BISH without choice.

The informal system BISH*

BISH* = BISH + inductive definitions with rules of countably many premises.

The measure theory in Bishop 1967 is bases on the inductive notion of Borel set.

The measure theory of Bishop-Bridges 1985 without inductive definitions.

Bishop's formalizations of Bishop 1967

A General Language ($\sim$1968/9): he developed a dependent type theory for BISH*, before Martin-Löf (unpublished).

Mathematics as a numerical language (1969): HA^ω, and he indicates how to compile his formalization into the programming language Algol.

How to compile mathematics into Algol ($\sim$1969): he indicates how to compile his type theory into Algol (unpublished).

The influence of BCM to the foundations of mathematics

Formal systems for Bishop 1967

1. **Martin-Löf type theory** (Bishop sets-setoids, equality in sets-propositional equality)
2. **Constructive set theory** (Myhill: CST and CST*, Friedman, Aczel and Rathjen: CZF + DC, CZF + DC + REA)
3. **Feferman's explicit mathematics** (Feferman, Jäger).
4. HA^ω, the only system that existed before Bishop 1967 (Bishop, Goodman and Myhill)

How Bishop succeeded not to contradict with classical mathematics

Uniform continuity theorem

UCT: If $f : [a, b] \to \mathbb{R}$ is continuous, then it is uniformly continuous.

It holds in Brouwer's intuitionistic mathematics (INT) (proof based on fan theorem and the non-classical continuity principle).

It holds in classical mathematics (CLASS).

It is false in constructive recursive mathematics (RUSS). Based on Specker's theorem (there is an increasing sequence of rational numbers in $[0, 1]$ that does not converge to any real number), there is continuous $f : [0, 1] \to (0, 1)$ that is not uniformly continuous.

Bishop's way out: forget about pointwise continuous function. A continuous function $f : [a, b] \to \mathbb{R}$ is by definition uniformly continuous. A function $f : \mathbb{R} \to \mathbb{R}$ is continuous, if it continuous on every interval $[-n, n]$.

The great achievements of Bishop

1. BISH is in the common territory of INT, CLASS and RUSS.

A proof of a formula A in BISH generates a proof of the interpretation of A in INT, CLASS and RUSS.

2. He showed that one can reconstruct large parts of mathematics in BISH in a way recognizable to any mathematician.

He succeeded there where Brouwer failed.

The fundamental thesis of constructivism

How can there be numbers that are not computable by any known method? Does that not contradict the very essence of the concept of number, which is concerned with computation?

$$\mathrm{Goldbach}(n) \equiv n \text{ is the sum of two primes},$$

$$n_1 \equiv \begin{cases} 0 & , \forall_{n \in \mathbb{N}} \Big((4 \leq n \leq 10^2 \ \& \ \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big) \\ 1 & , \text{otherwise} \end{cases}$$

$$n_2 \equiv \begin{cases} 0 & , \forall_{n \in \mathbb{N}} \Big((4 \leq n \leq 10^{100} \ \& \ \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big) \\ 1 & , \text{otherwise} \end{cases}$$

$$n_3 \equiv \begin{cases} 0 & , \forall_{n \in \mathbb{N}} \Big((4 \leq n \ \& \ \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big) \\ 1 & , \text{otherwise} \end{cases}$$

With some patience a mathematician can compute n_1, and, possibly with the help of some computing machine, he can, in principle, compute n_2.

There is no known finite, purely routine, process to convert n_3 to canonical form. The provability of the formula

$$\forall_{n \in \mathbb{N}} \Big((4 \leq n \ \& \ \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big)$$

is known as the **Goldbach conjecture**, which is one of the oldest and best-known unsolved problems in number theory and all of mathematics.

The current computing machines have verified the Goldbach conjecture up to 4×10^{18}.

PEM, in general has no computational content (GC is true or false, but we cannot compute the value of n_3)!

A representation m of a natural number is called **real**, if it can be converted, *in principle*, to a canonical natural number m^* by a finite, purely routine, process.

Fundamental thesis of constructivism: Only real representations of natural numbers are accepted constructively.

The thesis is extended to real only representations of functions of type $\mathbb{N} \to \mathbb{N}$ and to real representations of rational numbers.

Canonicity property of Martin-Löf intensional type theory.

The real numbers under the fundamental thesis

A **real number** x is a **regular** sequence $x : \mathbb{N}^+ \to \mathbb{Q}$ i.e.,

$$\forall_{n,m \in \mathbb{N}^+} \left(|x_m - x_n| \leq \frac{1}{m} + \frac{1}{n} \right).$$

$$x =_{\mathbb{R}} y \equiv \forall_{n \in \mathbb{N}^+} \left(|x_n - y_n| \leq \frac{2}{n} \right).$$

The relation $=_{\mathbb{R}}$ is an equivalence relation on $\mathbb{R}$.

$$x \text{ is strictly positive } \equiv \exists_{n \in \mathbb{N}^+} \left(x_n > \frac{1}{n} \right)$$

$$x \text{ is positive } \equiv \forall_{n \in \mathbb{N}^+} \left(x_n \geq -\frac{1}{n} \right)$$

The definition of a real number differs from the classical one, as classically a real number is the equivalence class of the reals, as defined here, with respect to the equivalence relation of their equality.

The avoidance of equivalence classes is a central feature of Bishop-style constructive mathematics.

The **Royden number** is the sequence $\varrho : \mathbb{N}^+ \to \mathbb{Q}$, where,

$$\varrho_n \equiv \sum_{k=1}^{n} \frac{a_k}{2^k},$$

$$a_k = \begin{cases} 0 & , \forall_{n \in \mathbb{N}} \Big((4 \le n \le k \;\&\; \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big) \\ 1 & , \text{otherwise} \end{cases}$$

Proposition

The following hold:

(i) ϱ is a real number.

(ii) $\varrho \ge 0$.

(iii) If there is a proof of the disjunction

$$\varrho > 0 \;\vee\; \varrho = 0,$$

then the Goldbach conjecture is decided i.e., there is a proof of the Goldbach conjecture or a proof of the negation of the Goldbach conjecture.

The previous result explains why we gave a separate definition of $x \geq 0$ and didn't define it as the disjunction $x > 0 \vee x = 0$.

Corollary

If there is a proof of the disjunction

$$\varrho > 0 \ \vee \ \varrho = 0 \ \vee \varrho < 0,$$

then the Goldbach conjecture is decided.

Hence the classical trichotomy

$$x < y \ \vee \ x = y \ \vee x > y$$

cannot be accepted, the following property is its constructive alternative.

Proposition

If $x, y, z \in \mathbb{R}$ such that $x < y$, then

$$x < z \ \vee \ z < y.$$

Proposition

The **modified Royden number** ϱ^* *is defined by*

$$\varrho^* \equiv \sum_{k=1}^{\infty} \frac{a_{2k}}{(-2)^k},$$

$$a_{2k} = \begin{cases} 0 & , \forall_{n \in \mathbb{N}} \Big((4 \le n \le 2k \ \& \ \mathrm{Even}(n)) \Rightarrow \mathrm{Goldbach}(n) \Big) \\ 1 & , \textit{otherwise} \end{cases}$$

(i) $\varrho^* \in \mathbb{R}$*.*
(ii) If there is a proof of the disjunction

$$\varrho^* \ge 0 \ \vee \ \varrho^* \le 0,$$

then the Goldbach conjecture is decided.

Corollary

Consider the following equation (E):

$$x(x-\varrho^*)=0.$$

(i) The real number $0 \wedge \varrho^*$ is a solution of (E).
(ii) If there is a proof of the disjunction

$$(0 \wedge \varrho^*)=0 \ \vee \ (0 \wedge \varrho^*)=\varrho^*,$$

then the Goldbach conjecture is decided.
(iii) If there is a proof of the implication

$$x(x-\varrho^*)=0 \Rightarrow (x=0 \ \vee \ x=\varrho^*),$$

then the Goldbach conjecture is decided.

Let $\mathbb{Q}^*[x] \equiv \mathbb{Q}[x] \setminus \{\bar{0}\}$. The set of **algebraic** real numbers $\mathbb{A}$ is defined by

$$\mathbb{A} \equiv \{x \in \mathbb{R} \mid \exists_{f \in \mathbb{Q}^*[x]}(f(x) = 0)\}.$$

Theorem (Julian, Mines, Richman)

If $a, b \in \mathbb{A}$, then

$$a < b \ \vee \ a = b \ \vee a > b.$$

In CLASS one finds important theorems in disjunctive form for which no method is known (yet) that decides which disjunct is the case. E.g., Jensen proved in the early 70's that the universe of sets V is either "very close" to Gödel's constructible universe L, which is an inner model of Zermelo-Fraenkel axiomatic set theory ZF in which the axiom of choice and the generalised continuum hypothesis are true in it, or "very far" from it.

Theorem

Exactly one of the following hold:

(i) Every singular cardinal γ is singular in L, and $(\gamma^+)^L = \gamma^+$.
(ii) Every uncountable cardinal is inaccessible in L.

Note that the proof of this theorem cannot specify which one of the two cases holds. The existence of large cardinals implies (ii), but this existence is unprovable in ZFC

A similar dichotomy for the inner model HOD (hereditarily ordinal definable sets) was proved by Woodin a few years ago.

Assuming the existence of an extendible cardinal, the first alternative of Woodin's dichotomy implies that HOD is close to V, and the second that HOD is far from V.

At the moment there is no evidence which one of the two alternatives is the right one, a fact with important consequences for the future of set theory.

Bishop spaces

2 difficulties in constructivising general topology

Bishop 1973: *The constructivisation of general topology is impeded by* **two obstacles**.

First, *the classical notion of a topological space is not constructively viable.*

Second, *even for metric spaces the classical notion of a continuous function is not constructively viable; the reason is that there is no constructive proof that a (pointwise) continuous function from a compact (complete and totally bounded) metric space to* $\mathbb{R}$ *is uniformly continuous.*

Since uniform continuity for functions on a compact space is the useful concept, pointwise continuity (no longer useful for proving uniform continuity) is left with no useful function to perform. Since uniform continuity cannot be formulated in the context of a general topological space, the latter concept also is left with no useful function to perform.

What if

there is a constructive notion $\mathcal{F}$ of an abstract topological space which does not copy or follow the pattern of the classical topological space, and at the same time

what if there is a constructive notion h of a "continuous" function between two such spaces $\mathcal{F}$ and $\mathcal{G}$ such that although uniform continuity is not part of the definition of this notion, in many expected cases it is reduced to uniform continuity.

Then we can hope to overcome the 2 difficulties in the constructivisation of topology.

$\mathcal{F} =$ **Bishop space**,
$h =$ **Bishop morphism**.

There are such reducibility results, which indicate that

a Bishop morpmism is an abstract notion of uniform continuity.

Other approaches to constructive topology

1. Intuitionistic topology (**Brouwer, Freudenthal, Troelstra, Waaldjik**).
2. Grayson's direct study of the axioms of topology using intuitionistic logic.
3. Formal topology (**Martin-Löf, Sambin**): a constructive and predicative generalization of the theory of frames and locales.
4. The theory of Bishop's neighborhood spaces (**Ishihara**): points are accepted, it is within BISH and its concepts are positively defined.
5. The theory of apartness spaces (**Bridges, Vîță**): points are accepted and it is within BISH.

3: It is point-free.
3-4: The notion of a base is central.
1-4: The topological structure "mimics" the classical one.
5: The topological structure does not "mimic" the classical one.
1-5: The topological structure comes first and the continuous functions are defined a posteriori.

The short history of the subject

1. Bishop introduced function spaces, here called Bishop spaces, in 1967.
2. Myhill commented on them in his 1975-paper.
3. Bridges revived the subject in 2012.
4. Directly after that Ishihara studied the relation of the subcategory **Fun** of the category **Bis** of Bishop spaces with the category of neighborhood spaces **Nbh** (2013).
5. We showed that we can develop non-trivial parts of general topology within TBS.

The main characteristics of TBS

1. The theory of Bishop spaces (TBS) is an approach to constructive point-function topology.
2. Points are accepted from the beginning, hence it is not a point-free approach to topology.
3. Most of its notions are function-theoretic. Set-theoretic notions are avoided or play a secondary role to its development.
4. It is constructive. We work within Bishop's informal system of constructive mathematics BISH, inductive definitions with rules of countably many premises included.
5. It has simple foundation and it follows the style of standard mathematics.

Continuity as a primitive notion

A **Bishop space** is a pair $\mathcal{F} = (X, F)$, where X is an inhabited set and $F \subseteq \mathbb{F}(X)$, a **Bishop topology**, or simply a **topology**, satisfies the following conditions:

(BS_1) $a \in \mathbb{R} \to \overline{a} \in F$.
(BS_2) $f \in F \to g \in F \to f + g \in F$.
(BS_3) $f \in F \to \phi \in \mathrm{B}(\mathbb{R}) \to \phi \circ f \in F$,
(BS_4) $f \in \mathbb{F}(X) \to U(F, f) \to f \in F$.

$$U(g, f, \epsilon) := \forall_{x \in X}(|g(x) - f(x)| \leq \epsilon),$$

$$U(\Phi, f) := \forall_{\epsilon > 0}\exists_{g \in \Phi}(U(g, f, \epsilon)).$$

$$fg, \lambda f, -f, f \vee g, f \wedge g, |f| \in F$$

$$\mathrm{Const}(X) \subseteq F \subseteq \mathbb{F}(X)$$

A **morphism** from $\mathcal{F}$ to $\mathcal{G}$ is a function $h : X \to Y$ such that

$$\forall_{g \in G}(g \circ h \in F).$$

$\mathrm{Mor}(\mathcal{F}, \mathcal{G})$ is the set of the morphisms from $\mathcal{F}$ to $\mathcal{G}$.
$F = \mathrm{Mor}(\mathcal{F}, \mathcal{R})$, $\mathcal{R} = (\mathbb{R}, \mathrm{B}(\mathbb{R}))$ is the **Bishop space of reals**.

$$\frac{f_0 \in F_0}{f_0 \in \mathcal{F}(F_0)} \quad \frac{a \in \mathbb{R}}{\overline{a} \in \mathcal{F}(F_0)} \quad \frac{f, g \in \mathcal{F}(F_0)}{f + g \in \mathcal{F}(F_0)},$$

$$\frac{f \in \mathcal{F}(F_0),\ \phi \in \mathrm{B}(\mathbb{R})}{\phi \circ f \in \mathcal{F}(F_0)} \quad \frac{(g \in \mathcal{F}(F_0),\ U(g, f, \epsilon))_{\epsilon > 0}}{f \in \mathcal{F}(F_0)},$$

$$\frac{g_1 \in \mathcal{F}(F_0)\ \wedge\ U(g_1, f, \frac{1}{2}),\ g_2 \in \mathcal{F}(F_0)\ \wedge\ U(g_2, f, \frac{1}{2^2}),\ g_3 \in \mathcal{F}(F_0)\ \wedge}{f \in \mathcal{F}(F_0)}$$

$$\begin{aligned}
&\forall_{f_0 \in F_0}(P(f_0)) \to \\
&\forall_{a \in \mathbb{R}}(P(\overline{a})) \to \\
&\forall_{f, g \in \mathcal{F}(F_0)}(P(f) \to P(g) \to P(f + g)) \to \\
&\forall_{f \in \mathcal{F}(F_0)} \forall_{\phi \in \mathrm{B}(\mathbb{R})}(P(f) \to P(\phi \circ f)) \to \\
&\forall_{f \in \mathcal{F}(F_0)}(\forall_{\epsilon > 0} \exists_{g \in \mathcal{F}(F_0)}(P(g)\ \wedge\ U(g, f, \epsilon)) \to P(f)) \to \\
&\forall_{f \in \mathcal{F}(F_0)}(P(f)).
\end{aligned}$$

Lifting of morphisms: If $\mathcal{G} = (Y, \mathcal{F}(G_0))$, then $h : X \to Y \in \mathrm{Mor}(\mathcal{F}, \mathcal{G})$ if and only if $\forall_{g_0 \in G_0}(g_0 \circ h \in F)$.

Theorem (Uniform continuity theorem for morphisms)

If $a, b \in \mathbb{R}$ such that $a < b$, then $f : [a, b] \to \mathbb{R} \in \mathrm{Mor}(\mathcal{R}_{|[a,b]}, \mathcal{R})$ if and only if f is uniformly continuous on $[a, b]$.

Theorem

If X and Y are compact metric spaces, then $h : X \to Y \in \mathrm{Mor}(\mathcal{U}(X),\mathcal{U}(Y))$ if and only if h is uniformly continuous.

Bridges's forward uniform continuity theorem within TBS

Theorem

Suppose that X is a compact metric space, Y is a metric space, and $h : X \to Y$ such that $\forall_{g \in U_0(Y)}(g \circ h \in C_u(X))$. Then the following hold:

(i) h is pointwise continuous and $h(X)$ is bounded.
(ii) h is uniformly continuous if and only if $h(X)$ is totally bounded.
(iii) If Y is locally compact, then h is uniformly continuous.

Theorem (Forward uniform continuity theorem, BISH + AS)

Suppose that X is a compact metric space, Y is a metric space and $h : X \to Y$. If $\forall_{g \in U_0(Y)}(g \circ h \in C_u(X))$, then h is uniformly continuous i.e.,

$$h \in \mathrm{Mor}(\mathcal{U}(X), \mathcal{C}_0(Y)) \Rightarrow h \text{ is uniformly continuous.}$$

The converse holds trivially, since d_y is uniformly continuous.
Bridges 1976: (FUCT) "*a possible constructive substitute for* UCT".

The future of BCM

BCM today: Munich, JAIST (Ishihara), Cristchurch (Bridges), Stockholm (Palmgren), Gothenburg (Coquand).

I. Constructive reverse mathematics.

II. Applications to physics, financial mathematics, constructive operations research etc.

III. Handbook of Bishop's constructive mathematics.

IV. Many interactions between BISH and Martin-Löf type theory and HoTT.

V. Implementation of results in BISH in proof assistants like Coq, Agda, Minlog etc.

- M. J. Beeson: *Foundations of Constructive Mathematics*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Springer Verlag, 1985.
- E. Bishop: *Foundations of Constructive Analysis*, McGraw-Hill, 1967.
- E. Bishop: A General Language, unpublished manuscript, 1969?
- E. Bishop: How to Compile Mathematics into Algol, unpublished manuscript, 1969?
- E. Bishop: Aspects of Constructivism, Lecture notes, 1972.
- E. Bishop and D. S. Bridges: *Constructive Analysis*, Grundlehren der Math. Wissenschaften 279, Springer-Verlag, Heidelberg-Berlin-New York, 1985.
- P. Martin-Löf: An intuitionistic theory of types: predicative part, in H. E. Rose and J. C. Shepherdson (Eds.) *Logic Colloquium'73*, pp.73-118, North-Holland, 1975.