On the Formal Constructive Theory of Computable Functionals TCF$^+$

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Abstract

Our aim is to present the current stage of formation of the constructive formal theory of computable functionals TCF$^+$.

While the idea of a formal theory of computable functionals based on the partial continuous functionals as its intended domain goes back to Scott's LCF [Scott 1969/1993], Schwichtenberg's formal theory TCF [Schwichtenberg and Wainer 2012], a common extension of Plotkin's PCF and Gödel's system **T**, uses, in contrast to LCF, non-flat free algebras as semantical domains for the base types. These algebras are given by their constructors, which can be proved in TCF to be injective with disjoint ranges. Moreover, the underlying logic of TCF is minimal.

The passage from TCF to TCF$^+$ is forced by our need to have a formal theory better adjusted to the intended model. Since a partial continuous functional of a type ρ over some base algebras is an ideal of the corresponding information system C_ρ, we would like to represent within our formal theory not only the functionals themselves but also their finite approximations, i.e., tokens and formal neighborhoods contained in them. The system TCF$^+$ is such a formal theory first developed in [Huber et al. 2010].

We present an updated version of TCF$^+$ and of the proofs within TCF$^+$ of a generalization of Kreisel's density theorem and Plotkin's definability theorem. We also point to new case studies that could be examined within TCF$^+$.

Selected References

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